The past decade has seen many advances in physical-layer wireless communication theory and their implementation in wireless systems. This textbook takes a unified view of the fundamentals of wireless communication and explains the web of concepts underpinning these advances at a level accessible to an audience with a basic background in probability and digital communication. Topics covered include MIMO (multiple input multiple output) communication, space-time coding, opportunistic communication, OFDM and CDMA. The concepts are illustrated using many examples from wireless systems such as GSM, IS-95 (CDMA), IS-856 (1× EV-DO), Flash OFDM and ArrayComm SDMA systems. Particular emphasis is placed on the interplay between concepts and their implementation in systems. An abundant supply of exercises and figures reinforce the material in the text. This book is intended for use on graduate courses in electrical and computer engineering and will also be of great interest to practicing engineers.

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Fundamentals of Wireless Communication

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and

Pramod Viswanath
University of Illinois, Urbana-Champaign
To my family and Lavinia
   DT

To my parents and to Suma
   PV
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Preface

Why we wrote this book

The writing of this book was prompted by two main developments in wireless communication in the past decade. First is the huge surge of research activities in physical-layer wireless communication theory. While this has been a subject of study since the sixties, recent developments such as opportunistic and multiple input multiple output (MIMO) communication techniques have brought completely new perspectives on how to communicate over wireless channels. Second is the rapid evolution of wireless systems, particularly cellular networks, which embody communication concepts of increasing sophistication. This evolution started with second-generation digital standards, particularly the IS-95 Code Division Multiple Access standard, continuing to more recent third-generation systems focusing on data applications. This book aims to present modern wireless communication concepts in a coherent and unified manner and to illustrate the concepts in the broader context of the wireless systems on which they have been applied.

Structure of the book

This book is a web of interlocking concepts. The concepts can be structured roughly into three levels:

1. channel characteristics and modeling;
2. communication concepts and techniques;
3. application of these concepts in a system context.

A wireless communication engineer should have an understanding of the concepts at all three levels as well as the tight interplay between the levels. We emphasize this interplay in the book by interlacing the chapters across these levels rather than presenting the topics sequentially from one level to the next.
Preface

Chapter 2: basic properties of multipath wireless channels and their modeling (level 1).

Chapter 3: point-to-point communication techniques that increase reliability by exploiting time, frequency and spatial diversity (2).

Chapter 4: cellular system design via a case study of three systems, focusing on multiple access and interference management issues (3).

Chapter 5: point-to-point communication revisited from a more fundamental capacity point of view, culminating in the modern concept of opportunistic communication (2).

Chapter 6: multiuser capacity and opportunistic communication, and its application in a third-generation wireless data system (3).

Chapter 7: MIMO channel modeling (1).

Chapter 8: MIMO capacity and architectures (2).

Chapter 9: diversity–multiplexing tradeoff and space-time code design (2).

Chapter 10: MIMO in multiuser channels and cellular systems (3).

How to use this book

This book is written as a textbook for a first-year graduate course in wireless communication. The expected background is solid undergraduate/beginning graduate courses in signals and systems, probability and digital communication. This background is supplemented by the two appendices in the book. Appendix A summarizes some basic facts in vector detection and estimation in Gaussian noise which are used repeatedly throughout the book. Appendix B covers the underlying information theory behind the channel capacity results used in this book. Even though information theory has played a significant role in many of the recent developments in wireless communication, in the main text we only introduce capacity results in a heuristic manner and use them mainly to motivate communication concepts and techniques. No background in information theory is assumed. The appendix is intended for the reader who wants to have a more in-depth and unified understanding of the capacity results.

At Berkeley and Urbana-Champaign, we have used earlier versions of this book to teach one-semester (15 weeks) wireless communication courses. We have been able to cover most of the materials in Chapters 1 through 8 and parts of 9 and 10. Depending on the background of the students and the time available, one can envision several other ways to structure a course around this book. Examples:

- A senior level advanced undergraduate course in wireless communication: Chapters 2, 3, 4.
- An advanced graduate course for students with background in wireless channels and systems: Chapters 3, 5, 6, 7, 8, 9, 10.
Preface

- A short (quarter) course focusing on MIMO and space-time coding: Chapters 3, 5, 7, 8, 9.

The more than 230 exercises form an integral part of the book. Working on at least some of them is essential in understanding the material. Most of them elaborate on concepts discussed in the main text. The exercises range from relatively straightforward derivations of results in the main text, to “back-of-envelope” calculations for actual wireless systems, to “get-your-hands-dirty” MATLAB types, and to reading exercises that point to current research literature. The small bibliographical notes at the end of each chapter provide pointers to literature that is very closely related to the material discussed in the book; we do not aim to exhaust the immense research literature related to the material covered here.
Acknowledgements

We would like first to thank the students in our research groups for the selfless help they provided. In particular, many thanks to: Sanket Dusad, Raúl Etkin and Lenny Grokop, who between them painstakingly produced most of the figures in the book; Aleksandar Jovičić, who drew quite a few figures and proofread some chapters; Ada Poon whose research shaped significantly the material in Chapter 7 and who drew several figures in that chapter as well as in Chapter 2; Saurabha Tavildar and Lizhong Zheng whose research led to Chapter 9; Tie Liu and Vinod Prabhakaran for their help in clarifying and improving the presentation of Costa precoding in Chapter 10.

Several researchers read drafts of the book carefully and provided us with very useful comments on various chapters of the book: thanks to Stark Draper, Atilla Eryilmaz, Irem Koprulu, Dana Porrat and Pascal Vontobel. This book has also benefited immensely from critical comments from students who have taken our wireless communication courses at Berkeley and Urbana-Champaign. In particular, sincere thanks to Amir Salman Avestimehr, Alex Dimakis, Krishnan Eswaran, Jana van Greunen, Nils Hoven, Shridhar Mubaraq Mishra, Jonathan Tsao, Aaron Wagner, Hua Wang, Xinzhou Wu and Xue Yang.

Earlier drafts of this book have been used in teaching courses at several universities: Cornell, ETHZ, MIT, Northwestern and University of Colorado at Boulder. We would like to thank the instructors for their feedback: Helmut Bölcskei, Anna Scaglione, Mahesh Varanasi, Gregory Wornell and Lizhong Zheng. We would like to thank Ateet Kapur, Christian Peel and Ulrich Schuster from Helmut’s group for their very useful feedback. Thanks are also due to Mitchell Trott for explaining to us how the ArrayComm systems work.

This book contains the results of many researchers, but it owes an intellectual debt to two individuals in particular. Bob Gallager’s research and teaching style have greatly inspired our writing of this book. He has taught us that good theory, by providing a unified and conceptually simple understanding of a morass of results, should shrink rather than grow the knowledge tree. This book is an attempt to implement this dictum. Our many discussions with...
Acknowledgements

Rajiv Laroia have significantly influenced our view of the system aspects of wireless communication. Several of his ideas have found their way into the “system view” discussions in the book.

Finally we would like to thank the National Science Foundation, whose continual support of our research led to this book.
Notation

Some specific sets
\( \mathcal{R} \) Real numbers
\( \mathcal{C} \) Complex numbers
\( S \) A subset of the users in the uplink of a cell

Scalars
\( m \) Non-negative integer representing discrete-time
\( L \) Number of diversity branches
\( \ell \) Scalar, indexing the diversity branches
\( K \) Number of users
\( N \) Block length
\( N_c \) Number of tones in an OFDM system
\( T_c \) Coherence time
\( T_d \) Delay spread
\( W \) Bandwidth
\( n_t \) Number of transmit antennas
\( n_r \) Number of receive antennas
\( n_{\text{min}} \) Minimum of number of transmit and receive antennas
\( h[m] \) Scalar channel, complex valued, at time \( m \)
\( h^* \) Complex conjugate of the complex valued scalar \( h \)
\( x[m] \) Channel input, complex valued, at time \( m \)
\( y[m] \) Channel output, complex valued, at time \( m \)
\( \mathcal{N}(\mu, \sigma^2) \) Real Gaussian random variable with mean \( \mu \) and variance \( \sigma^2 \)
\( \mathcal{CN}(0, \sigma^2) \) Circularly symmetric complex Gaussian random variable: the real and imaginary parts are i.i.d. \( \mathcal{N}(0, \sigma^2/2) \)
\( N_0 \) Power spectral density of white Gaussian noise
\( \{w[m]\} \) Additive Gaussian noise process, i.i.d. \( \mathcal{CN}(0, N_0) \) with time \( m \)
\( z[m] \) Additive colored Gaussian noise, at time \( m \)
\( P \) Average power constraint measured in joules/symbol
\( \bar{P} \) Average power constraint measured in watts
SNR Signal-to-noise ratio
SINR Signal-to-interference-plus-noise ratio
## List of notation

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<td>$\varepsilon_b$</td>
<td>Energy per received bit</td>
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<td>$P_e$</td>
<td>Error probability</td>
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### Capacities
- $C_{\text{awgn}}$ Capacity of the additive white Gaussian noise channel
- $C_\epsilon$ $\epsilon$-Outage capacity of the slow fading channel
- $C_{\text{sum}}$ Sum capacity of the uplink or the downlink
- $C_{\text{sym}}$ Symmetric capacity of the uplink or the downlink
- $C_{\text{sym}}(\epsilon)$ $\epsilon$-Outage symmetric capacity of the slow fading uplink channel
- $P_{\text{out}}$ Outage probability of a scalar fading channel
- $P_{\text{out}}(\text{Ala})$ Outage probability when employing the Alamouti scheme
- $P_{\text{out}}(\text{rep})$ Outage probability with the repetition scheme
- $P_{\text{out}}(\text{ul})$ Outage probability of the uplink
- $P_{\text{out}}(\text{mimo})$ Outage probability of the MIMO fading channel
- $P_{\text{out}}(\text{ul-mimo})$ Outage probability of the uplink with multiple antennas at the base-station

### Vectors and matrices
- $\mathbf{h}$ Vector, complex valued, channel
- $\mathbf{x}$ Vector channel input
- $\mathbf{y}$ Vector channel output
- $\mathcal{CN}(0, \mathbf{K})$ Circularly symmetric Gaussian random vector with mean zero and covariance matrix $\mathbf{K}$
- $\mathbf{w}$ Additive Gaussian noise vector $\mathcal{CN}(0, N_0 \mathbf{I})$
- $\mathbf{h}^*$ Complex conjugate-transpose of $\mathbf{h}$
- $\mathbf{d}$ Data vector
- $\tilde{\mathbf{d}}$ Discrete Fourier transform of $\mathbf{d}$
- $\mathbf{K}$ Matrix, complex valued, channel
- $\mathbf{K}_{\text{cov}}$ Covariance matrix of the random complex vector $\mathbf{x}$
- $\mathbf{H}$ Complex conjugate-transpose of $\mathbf{H}$
- $\mathbf{H}^t$ Transpose of matrix $\mathbf{H}$
- $\mathbf{Q}$, $\mathbf{U}$, $\mathbf{V}$ Unitary matrices
- $\mathbf{I}_n$ Identity $n \times n$ matrix
- $\Lambda$, $\Psi$ Diagonal matrices
- $\text{diag}\{p_1, \ldots, p_n\}$ Diagonal matrix with the diagonal entries equal to $p_1, \ldots, p_n$
- $\mathbf{C}$ Circulant matrix
- $\mathbf{D}$ Normalized codeword difference matrix

### Operations
- $\mathbb{E}[x]$ Mean of the random variable $x$
- $\mathbb{P}[A]$ Probability of an event $A$
- $\text{Tr}[\mathbf{K}]$ Trace of the square matrix $\mathbf{K}$
- $\text{sinc}(t)$ Defined to be the ratio of $\sin(\pi t)$ to $\pi t$
- $\mathcal{Q}(a)$ $\int_a^{\infty} (1/\sqrt{2\pi}) \exp^{-x^2/2} \, dx$
- $\mathcal{L}(\cdot, \cdot)$ Lagrangian function