BOUNDARY CONFORMAL FIELD THEORY AND
THE WORLDSHEET APPROACH TO D-BRANES

Boundary conformal field theory is concerned with a class of two-dimensional quantum field theories that display a rich mathematical structure and have many applications, ranging from string theory to condensed matter physics. In particular, the framework allows discussion of strings and branes directly at the quantum level.

Written by internationally renowned experts, this comprehensive introduction to boundary conformal field theory reaches from theoretical foundations to recent developments, with an emphasis on the algebraic treatment of string backgrounds. Topics covered include basic concepts in conformal field theory with and without boundaries, the mathematical description of strings and D-branes, and the geometry of strongly curved spacetime. The book offers insights into string geometry that go beyond classical notions.

Describing the theory from basic concepts, and providing numerous worked examples from conformal field theory and string theory, this reference is of interest to graduate students and researchers in physics and mathematics.

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Boundary Conformal Field Theory
and the Worldsheet Approach to
D-Branes

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VOLKER SCHOMERUS
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To our parents

to Elena

und Hans Recknagel zum Angedenken
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