Chapter 1

**Reasoning, Communication and Connections in Mathematics: An Introduction**

Berinderjeet KAUR    TOH Tin Lam

This introductory chapter provides an overview of the chapters in the book. The chapters are organised according to three broad themes that are central to reasoning, communication and connections. The themes are mathematical tasks, classroom discourse and connections within and beyond mathematics. It ends with some concluding thoughts that readers may want to be cognizant of while reading the book and also using it for reference and further work.

1 Introduction

This yearbook of the Association of Mathematics Educators (A.M.E) in Singapore focuses on Reasoning, Communication and Connections in Mathematics. Like two of our past yearbooks, Mathematical Problem Solving (Kaur, Yeap, & Kapur, 2009) and Mathematical Applications and Modelling (Kaur & Dindyal, 2010), the theme of this book is also shaped by the framework of the school mathematics curriculum in Singapore, shown in Figure 1. The primary goal of school mathematics in Singapore is mathematical problem solving and amongst the processes specified explicitly for nurturing problem solvers are reasoning, communication and connections. In elaborating the framework, for both the primary and secondary students, the Ministry of Education (M.O.E) (2006a, 2006b), syllabus documents clarify that:
Mathematical reasoning refers to the ability to analyse mathematical situations and construct logical arguments. It is a habit of mind that can be developed through the application of mathematics in different contexts. Communication refers to the ability to use mathematical language to express mathematical ideas and arguments precisely, concisely and logically. It helps students develop their own understanding of mathematics and sharpen their mathematical thinking. Connections refer to the ability to see and make linkages among mathematical ideas, between mathematics and other subjects, and between mathematics and everyday life. This helps students make sense of what they learn in mathematics. (Ministry of Education, 2006a, p.14; 2006b, p.5)

![Figure 1. Framework of the Singapore school mathematics curriculum](image)

The fourteen peer-reviewed chapters in this yearbook address various aspects of reasoning, communication and connections. All the chapters arose out of the keynote lectures and workshops conducted during the Mathematics Teachers Conference 2011 which was jointly organised by the Association of Mathematics Educators and the Mathematics and Mathematics Education Academic Group at the National Institute of Education in Singapore. The authors of the chapters
were asked to focus on evidence-based practices that school teachers can experiment in their lessons to bring about meaningful learning outcomes. In the following sections we briefly outline how the chapters contribute towards reasoning, communication and connections respectively.

2 Mathematical Tasks

Central to all mathematics lessons are mathematical tasks. A mathematical task is defined as a set of problems or a single complex problem that focuses students’ attention on a particular mathematical idea (Stein, Grover, & Henningsen, 1996). From the TIMSS Video Study (NCES, 2003), in which Australia, Czech Republic, Hong Kong, Japan, Netherlands, Switzerland, and the United States participated, it was found that students spent over 80% of their time in mathematics class working on mathematical tasks. According to Doyle (1988), “the work students do, defined in large measure by the tasks teachers assign, determines how they think about a curricular domain and come to understand its meaning” (p. 167). Hence different kinds of tasks lead to different types of instruction, which subsequently lead to different opportunities for students learning (Doyle, 1988).

Mathematical tasks with high cognitive demand often require students to make explicit their thinking. These tasks are necessary for the advancement of reasoning, communication and connections during lessons. In chapter 2, Rahim, Hogan, and Chan report research on the status of epistemic framing of mathematical tasks in secondary three mathematics classes in Singapore. The findings reported in the chapter indicate that teachers provided students with relatively substantial opportunities to acquire procedural and metacognitive knowledge but not conceptual and epistemic knowledge. As the intellectual quality of knowledge work depends substantially, although not exclusively, on the nature and quality of tasks that teachers’ provide their students to work with, it is apparent that teachers need knowledge of such tasks themselves.

Boston and Smith (2009) report that research has consistently indicated that teachers selection of instructional tasks is largely based on
lists of skills and concepts they need to cover. Textbooks are often the main source of such tasks (Doyle, 1983; Kaur, 2010). As noted by Valverde, Bianchi, Wolfe, Schimdt, and Houang (2002) textbooks are often good guides for content but lack emphasis on processes such as problem solving or communication. In this book there are several chapters that provide teachers with insights on how to modify textbook type of mathematical tasks into ones that incorporate reasoning and facilitate communication in the lesson. In chapter 3, Thompson presents a variety of specific and general strategies that may be used to modify typical textbook exercises to incorporate reasoning and communication into the primary mathematics classroom. In chapter 4, Kaur introduces primary teachers to four “What” strategies that may be used to create tasks from textbook questions or student work so as to advance reasoning and communication in the primary school mathematics classrooms. The strategies are: What number makes sense?, What’s wrong?, What if?, and What’s the question if you know the answer? In the chapter how one primary 1 teacher enacted the strategy "What’s the question if you know the answer" is also presented to demonstrate that even primary one students can participate in lessons that call for reasoning and communication provided they are given the opportunity to do so. In chapter 5, Thompson draws on a framework from textbook research and shows how typical algebra and statistics textbook type of exercises may be modified to engage secondary school students in reasoning and justification. The framework has six components of proof-related reasoning, viz-a-viz finding counterexamples, investigating conjectures, making conjectures, developing arguments, evaluating arguments, and correcting mistakes in reasoning.

Lew and Jang, show in chapter 6 that project-based LOGO programming activities may be used to improve reasoning, communication and problem solving skills of high ability Year 6 students. These activities appeared to provide students with opportunities to activate and promote reasoning strategies such as analogy, generalisation, progressive and critical thinking and debugging based on visualisation and empirical inference. In chapter 7 Toh illustrates with appropriate examples, from the Advanced Level Mathematics curriculum for Years 11 and 12, how reasoning, communication and connections may be
infused into the teaching of mathematics for advanced learners of mathematics. Toh demonstrates that for mathematical reasoning, elementary mathematical proofs and derivation of results by first principles are good opportunities to transcend procedural emphasis of traditional teaching to higher level mathematical reasoning. He also illustrates that connections can be achieved by several means such as facilitating students to connect across different mathematical ideas, across other disciplines and to daily life.

Lastly, in chapter 8, Lowrie uses assessment tasks to make the case that visual and spatial reasoning plays an important role in communicating mathematical ideas. Furthermore he emphasises that the increasing reliance on graphics in today’s society require students to acquire different spatial-reasoning skills so that they consider all the elements of a task, including specific features of a graphic and the surrounding text, when solving mathematical tasks.

3 Classroom Discourse

Mathematical tasks mediate classroom discourse between teachers and students, and between students and students. As stated in the Professional Standards for Teaching Mathematics (National Council of Teachers of Mathematics, 1991), “the discourse of a classroom – the ways of representing, thinking, talking, agreeing, disagreeing – is critical to what students learn about mathematics as a domain of human inquiry with characteristic ways of knowing” (p.34). Teacher questions are a key element of classroom discourse. Such questions provide students interactive opportunities for learning. Hogan, Rahim, Chan, Kwek and Towndrow in chapter 9 report research on the kind of teacher questions that teachers focus on in mathematics lessons and the relationships between different kinds of teacher questions. Their research is based on survey data from students in secondary three mathematics classes in Singapore. The findings of the research show that there is a relatively high prevalence of performative questions arising from the IRE [teacher initiate (I), student respond (R), and teacher evaluate (E)] talk structure in the mathematics classes. This suggests that mathematics classrooms in
Singapore provide limited opportunity for students to engage in rich classroom conversations.

Analogy reasoning draws on pre-existing knowledge to make relational and structural similarity in objects and construct mathematical knowledge. Lee in Chapter 10 shows that analogy reasoning can facilitate rich discourse, both teacher initiated and student initiated, between the teacher and students. Student errors and misconceptions are often outcomes of classroom discourse where students fail to clarify their thinking or teachers adopt “recipe like” procedures that are not conceptually sound. Pang and Dindyal in chapter 10 studied students’ reasoning errors in proofs by mathematical induction. The errors made by the Year 12 students mainly arose from their inability to grasp that in the mathematical statement $P(n)$ $n$ must be an integer and that the set of numbers generalised by $n$ can be ordered. Also, many students regard the induction step solely as proving the $P(k + 1)$ statement and not proving that “if $P(k)$ is true then $P(k + 1)$ is true”. It is apparent that the errors were a result of perception that mathematical induction is a procedure and not a coherent deductive system of logical steps of a proof.

4 Connections Within and Beyond Mathematics

Among the aims of mathematics education in Singapore schools are:

- Acquire the necessary mathematical concepts and skills for everyday life, and for continuous learning in mathematics and related disciplines;
- Recognise and use connections among mathematical ideas, and between mathematics and other disciplines.

(Ministry of Education, 2006a, p. 2; 2006b, p. 11).

It is apparent from the above that connections within and beyond mathematics are an essential part of the school curriculum. Teachers must provide students with opportunities to experience connections in the mathematics they learn. This is possible through links between conceptual and procedural knowledge, connections among mathematical
topics and equivalent representations of the same concept (Coxford, 1995). Similarly, teachers must also provide students with opportunities to experience connections between mathematics and other disciplines of the school curriculum and daily life needs.

Leong in chapter 12 provides a portrait of mathematics classroom instructional practice in which mathematics is presented as skills and concepts that are closely connected to each other. Here, students are encouraged to see school mathematics as situated and connected to a wider context of mathematical knowledge. Leong discusses ways in which this alternative portrait of mathematics instruction can be enacted, drawing upon examples that include findings from a research project. In school education, numeracy is a fundamental component of learning, discourse and critique across all areas of the curriculum. In chapter 13, Kissane discusses the nature of numeracy and some recent work in the Australian primary and secondary curriculum that involves numeracy. According to Kissane, at the primary levels, situations that demand mathematical thinking arise in all aspects of the curriculum and thus provide opportunities for numeracy to be developed and at the secondary levels, mathematics is important for learning in other subjects.

Kemp in chapter 14, asserts that mathematics teachers often find that resources for teaching and learning, such as standard textbooks, do not connect well to the everyday worlds of students in their classes. In the chapter, Kemp shows that there are many opportunities in the classroom to make connections between school mathematics and the everyday world using the example of health. Lastly, Aslaksen in chapter 15 shows how astronomy and culture may be connected to mathematics learning in classrooms of students from primary schools to university.

4 Some Concluding Thoughts

The performance of students from Singapore in the Trends in International Mathematics and Science Study (TIMSS) 1995, 1999, 2003 and 2007 and the Programme for International Student Assessment (PISA) 2009 has been outstanding. This affirms that the basics of mathematical content knowledge are sound. However, the research
Reasoning, Communication and Connections in Mathematics

reported in this book on the epistemic framing of mathematical tasks and classroom talk in secondary three mathematics classes in Singapore show that the quality of knowledge work to facilitate mathematical practices in the classrooms were lacking. As, so aptly, stated by Ball (2003):

Mathematical practices involve more than what is normally thought of as mathematical knowledge. This area focusses on the mathematical know-how, beyond content knowledge, that constitutes expertise in learning and using mathematics. The term “practices” refers to the specific things that successful mathematics learners and user do. Justifying claims, using symbolic notation effectively, defining terms precisely, and making generalisations are examples of mathematical practices. Another example of mathematical practices is the way in which skilled mathematics users are able to model a situation to make it easier to understand and to solve problems related to it. Those skilled individuals might use algebraic notation cleverly to simplify a complex set of relationships, or they might recognise that a geometric representation makes a problem almost transparent, whereas the algebraic formulation, although correct, obscures it. (p. xviii)

Although, it is apparent from the Singapore school mathematics framework (shown in Figure 1) that the intended curricula places emphasis on processes such as reasoning, communication and connections, thinking skills and heuristics, and applications and modelling for the development of “practices” that Ball (2003) refers to, there appears to be lack of emphasis on these in lessons. To improve the quality of knowledge work to facilitate mathematical practices, teachers need to engage their students in rich mathematical tasks and classroom discourse.

The several chapters in the yearbook provide readers with ideas on how to improve the quality of knowledge work in mathematics classrooms through reasoning, communication and connections. We suggest you read the chapters and contextualise the ideas within the larger context of teaching and learning mathematics. We also like you to note that reasoning, communication and connections are not isolated
sub-components of Processes in the framework, rather they work in tandem with other components of the framework. For example, the use of appropriate questioning techniques, modification of textbook questions to engage students in reasoning and communication, use of visual and spatial reasoning for solving mathematical problems also help students regulate their learning and engage in reflection i.e. metacognition. Similarly, when teachers make connections across different mathematical ideas and disciplines of the school curriculum students get an opportunity to see the application and relevance of the mathematics they learn. This may help to nurture positive attitudes towards the learning of mathematics for some students.

Lastly, we like to offer a word of caution for readers who are often overwhelmed by yet more ideas in the book in view of their already content-heavy curriculum. We urge the readers to read the chapters carefully and try some of the ideas in their classrooms and convince themselves that these ideas offer a means of infusing reasoning, communication and connections in their lessons and engage students in meaningful mathematical practices.

References


