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Chapter 2
Traffic Grooming: Combinatorial Results and Practical Resolutions

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Abstract In an optical network using the wavelength division multiplexing (WDM) technology, routing a request consists of assigning it a route in the physical network and a wavelength. If each request uses \(1/g\) of the bandwidth of the wavelength, we will say that the grooming factor is \(g\). That means that on a given edge of the network we can groom (group) at most \(g\) requests on the same wavelength. With this constraint the objective can be either to minimize the number of wavelengths (related to the transmission cost) or minimize the number of Add/Drop Multiplexers (shortly ADM) used in the network (related to the cost of the nodes).

Here, we first survey the main theoretical results obtained for different grooming...
factors on various topologies: complexity, (in)approximability, optimal constructions, approximation algorithms, heuristics, etc. Then, we give an ILP formulation for multilayer traffic grooming and present some experimental results.

Key words: WDM networks, grooming, ADM, complexity, approximation algorithms, heuristics, integer linear programming

2.1 Introduction

Traffic grooming refers to techniques used to organize and simplify routing and switching in connection-oriented networks, such as WDM (wavelength division multiplexing) or MPLS (Multi-protocol Label Switching) networks, in order to improve the usage of the bandwidth and of the components, and therefore to reduce the network cost.

Typically, when establishing a connection in an optical network, one has to install some equipment at both extremities of the connection, that is, an optical transmitter (laser) at its source and an optical receiver at its destination. But due to the cost of building, installing, and maintaining devices, it is usually more interesting to use a single kind of device that can handle both transmission and reception. Such devices are called Light Termination Equipment, or LTE for short. Thus, every connection will involve two distinct LTEs, and two distinct connections may share the same LTE, provided that one ends at a node while the other starts from that same node. In this context, traffic grooming refers to minimizing the number of LTEs that are needed in the network to serve all connection requests. The problem of minimizing the number of LTEs in the network being NP-Hard [58, 84], research effort has concentrated on the development of efficient approximation algorithms for both static and online traffic [52, 59, 64, 65, 103]. This is the subject of Section 2.3.

At another level in the network, traffic grooming also refers to techniques used to combine low-speed traffic streams onto high speed wavelengths in order to minimize the network-wide cost in terms of electronic switching. Typically, nodes of the network insert and/or extract the data streams on a wavelength by means of add/drop multiplexers (ADMs). A WDM or DWDM (dense WDM) optical network can handle many wavelengths, each with large bandwidth available. On the other hand, a single user seldom needs such large bandwidth. Therefore, by using multiplexed access such as TDMA (time-division multiple access) or CDMA (code-division multiple access), different users can share the same wavelength, thereby optimizing the bandwidth usage of the network. By using traffic grooming, not only is the bandwidth usage optimized, but also the cost of the network can be cut by reducing the total number of ADMs. Such techniques become increasingly important for emerging network technologies, including SONET/WDM rings and MPLS/MPâS backbones [108], for which traffic grooming is essential.

In this context, one ADM is needed in a node each time we want to add or drop traffic to or from a wavelength. Therefore, one has to place one ADM in a node for
each wavelength in which traffic is added or dropped, as can be seen in Figure 2.1. Here, the bandwidth requirement of a traffic stream is expressed as a fraction of the bandwidth offered by a single wavelength, which we call the *grooming factor*, \( g \), and an ADM is able to drop (or add) up to \( g \) unitary traffic streams from (or to) a given wavelength. Thus, the traffic grooming problem is to minimize the total number of ADMs to be installed in the network in order to accommodate all traffic streams.

Given the general traffic grooming problem of minimizing the total number of ADMs to be installed in the network with respect to the traffic requirement being NP-complete \([21, 101]\), recent works focus on specific issues. Most of the algorithms aim at grooming traffic in such a way that all the traffic between any given pair of nodes is carried on a minimum number of wavelengths. However, a large part of the network cost depends on the capacity of the multiplexing equipment required at each node. Hence, in order to minimize the overall network cost, algorithms have to take into account a trade-off between the number of wavelengths used and the number of required ADMs. Indeed minimizing the number of ADMs is different from minimizing the number of wavelengths: the number of wavelengths and the number of ADMs cannot always be simultaneously minimized (see \([11, 21, 69]\) for unitary traffic). Both minimization problems have been considered by many authors. See, for example, the surveys \([3, 56]\) for minimization of the number of wavelengths and \([10, 69, 70, 73, 112, 115]\) for minimization of ADMs, and \([72, 81]\) for online approaches. Numerical results, heuristics, and tables might be found in \([11, 113]\), and extensions to multicast connection requests in \([51, 107]\).

The reader may also refer to the surveys \([27, 57, 89, 117]\) and books \([55, 106, 118]\) for other aspects of traffic grooming that are not considered here; in particular, *waveband switching* allows switching together a set of predetermined wavelengths (band) issued from one fiber and going to another \([18–20, 75, 116]\). Various other concepts might also been considered as traffic grooming, such as Lighttrails \([114]\), Lighttours \([105]\), or bus labeling \([16, 17]\).

In this chapter, we give an overview of the traffic grooming problems that have been addressed within the European project COST 293 GRAAL, and we survey the main exact and approximate results obtained so far for static and online traffic. We present practical approaches for multilayer traffic grooming. The results have been obtained using a large variety of mathematical tools including graph theory, design theory, linear programming, combinatorial optimization, and game theory.

This chapter is structured as follows. We start in Section 2.2 with a general definition of the traffic grooming problem, and we give some examples. In Section 2.3 we present the modelization and the main results obtained for minimizing the number of LTEs in a network. We continue in Section 2.4 with the more general model of minimizing the number of ADMs, for which we survey the main combinatorial results. Then, in Section 2.5, we present an efficient ILP model for multilayer traffic grooming on general networks subject to general traffic demands. We also present some experimental results. We finally conclude this chapter in Section 2.6.
2.2 Problem Definition and Examples

In this section, we first give precise descriptions and models of LTE and ADM, and then formalize the traffic grooming problem considered here.

A Light Termination Equipment, LTE, is a device that realizes the interface between the optical domain and the electronic domain. It is constituted of one optical receiver and one optical transmitter, so every connection involves two distinct LTEs, one at each endpoint. In this chapter, we assume that the receiver and the transmitter of an LTE are tuned on the same wavelength (other assumptions are possible). Also, two distinct connections may share an LTE, provided that one ends at a node while the other starts from that same node, and that both connections are assigned the same wavelength.

An Add/Drop Multiplexer, ADM, is a device used in synchronous transmission networks (SDHs or SONETs) to add (insert) or drop (remove) lower-data-rate channel traffic from the higher-rate aggregated channel. In optical networks, each ADM contains an LTE to realize the interface between the optical domain (high-speed channel) and the electronic domain (lower-speed channels). Thus, an ADM operates on a single high-speed data stream, and so a single wavelength, as can be seen in Figure 2.1. The cost of an ADM is given by its capacity, that is, the maximum number of low-speed channels (provided each of them has a unitary bandwidth requirement) that can be added or dropped from the wavelength. The capacity of an ADM is called the grooming factor or grooming ratio. Finally, note that with grooming factor 1, an ADM is nothing other than an LTE.

In optical networks with grooming capabilities, the traffic demands are expressed in terms of low-speed data channels. Thus, one has to assign to each connection request a path and a wavelength with the capacity constraint that at most $g$ (grooming factor) connection requests are assigned the same wavelength on the same link of the network.

An instance of the traffic grooming problem is a triple $(G,I,g)$ where $G = (V,E)$ is a graph modeling the network topology, $I$ is a set of connection requests, and $g$ is a positive integer, namely the grooming factor.
Given a connection request $r \in I$ identified by a couple of nodes aiming to communicate, let $P_r$ be the set of the paths in $G$ connecting the two endpoints relative to $r$. We have two main issues:

- the determination of a path system (or path assignment) of $(G, I)$, that is, a function $p : I \mapsto \bigcup_{r \in I} P_r$;
- the determination of a proper coloring (or wavelength assignment) of $(G, I)$, that is, a function $w : I \mapsto N^+ = \{1, 2, \ldots\}$ such that for any edge $e \in E$ at most $g$ paths using $e$ are colored with the same color.

Some of the results presented in this chapter deal with both issues (Section 2.5), while others, given a path system in the input, focus only on the determination of a proper coloring (Sections 2.3 and 2.4).

Every colored request $r \in I$ needs an ADM at each of its endpoint nodes. Following the above description of ADMs, given a grooming factor $g$, at most $g$ paths with the same color, incident to a node through the same edge, can use the same ADM. Furthermore, the same ADM can also be shared by at most $g$ paths with the same color, incident to the same node through another incident edge.

The traffic grooming problem is the optimization problem of finding a proper coloring $w$ of $(G, I, g)$ minimizing the total number of ADMs used. Let $A(G, I, g)$ be the optimal value for such a problem.

To establish ideas we now provide two examples, for uni- and bidirectional rings, respectively.

Unidirectional Ring

Suppose we have a unidirectional ring with four nodes $\{1, 2, 3, 4\}$ and an all-to-all unitary traffic (one request between each pair of nodes). Since we need one ADM at each extremity of a request, and the routing is unique, we can put requests $(i, j)$ and $(j, i)$ on the same wavelength, thus using $1/g$ of the capacity of that wavelength on the ring. We call such pair of symmetric requests a circle. There are therefore six circles $(i, j)$ for $1 \leq i < j \leq 4$. If there is no grooming (i.e., $g = 1$) we need six wavelengths (one per circle) and a total of 12 ADMs. If we have a grooming factor $g = 2$, we can put on the same wavelength two circles, using three or four ADMs according to whether they share an end node or not. For example, we can put together $(1, 2)$ and $(2, 3)$ on one wavelength; $(1, 3)$ and $(3, 4)$ on a second wavelength; and $(1, 4)$ and $(2, 4)$ on a third wavelength, for a total of nine ADMs, and this is optimal.

Now, if we allow a grooming factor $g = 3$, we can use only two wavelengths. If we put together on one wavelength $(1, 2)$, $(2, 3)$, and $(3, 4)$ and on the other $(1, 3)$, $(2, 4)$, and $(1, 4)$, we need eight ADMs (solution $a$, Figure 2(a)); but we can do better by putting on the first wavelength $(1, 2)$, $(2, 3)$ and $(1, 3)$ and on the second $(1, 4)$, $(2, 4)$ and $(3, 4)$, using seven ADMs (solution $b$, Figure 2(b)).

More formally, in the above example with $N = 4$ and $g = 3$, solution $a$ consists of a decomposition of $K_4$ (all circles) into two paths with four vertices each,
(a) Solution with eight ADMs. Circles (1,2), (2,3), and (3,4) on the first wavelength, and (1,3), (2,4), and (1,4) on the second wavelength.

(b) Solution with seven ADMs. Circles (1,2), (2,3), and (1,3) on the first wavelength, and (1,4), (2,4), and (3,4) on the second wavelength.

(c) Decomposition of $K_4$ associated with the two solutions. Each edge of $K_4$ corresponds to a circle.

Fig. 2.2 Traffic grooming for a unidirectional ring with four nodes, grooming factor $g = 3$ all-to-all unitary traffic. Solution 2(a) with eight ADMs, solution 2(b) with seven ADMs, and corresponding decompositions of $K_4$. 
[1, 2, 3, 4] and [1, 4, 2, 3], while solution b corresponds to a decomposition into a triangle (1, 2, 3) and a star with edges (1, 4), (2, 4), and (3, 4).

Bidirectional Ring

Consider now a bidirectional ring on five nodes \{0, 1, 2, 3, 4\} with all-to-all unitary traffic modeled by the complete symmetric digraph \(K_5^+\). In this setting, it is more interesting to route requests \((i, j)\) and \((j, i)\) on different wavelengths with shortest path routing. For example, with grooming factor \(g = 3\), we can put on a wavelength routed clockwise requests \((i, i + 1 \mod 5)\) and \((i, i + 2 \mod 5)\), and on a wavelength routed counterclockwise requests \((i, i - 1 \mod 5)\) and \((i, i - 2 \mod 5)\). We need five ADMs on each wavelength so overall ten ADMs. But if requests \((i, j)\) and \((j, i)\) are routed on a same wavelength, then we can put at most three circles (pairs of symmetric requests) per wavelength using at least three ADMs. Since \(K_5^+\) contains ten circles, we need four wavelengths, three of them with three circles and at least three ADMs and one of them with at least one circle and two ADMs, so overall 11 ADMs.

With grooming factor \(g = 2\), we can put on one wavelength requests \((i, i + 1 \mod 5)\) and on another wavelength requests \((i, i + 2 \mod 5)\). Symmetric requests are routed similarly in opposite direction and we obtain the partition of Figure 3(b) using overall 20 ADMs. But we can do better by putting on a first wavelength requests \((i, i + 1 \mod 5)\), \((0, 2)\) and \((2, 4)\) using five ADMs, and on a second wavelength requests \((1, 3)\), \((3, 5)\), and \((4, 1)\) using four ADMs. We obtain the partition of Figure 3(c) using overall \(2(5 + 4) = 18\) ADMs.

Fig. 2.3 Two valid partitions of \(K_5^+\) when \(g = 2\), using different number of ADMs. Symmetric requests are routed counterclockwise and partitioned similarly.
2.3 Minimizing the Usage of Light Termination Equipment

In this section, we concentrate on the traffic grooming problem of minimizing the total number of LTEs that are needed in the network to serve all connection requests. This problem is NP-hard [58, 84] in general but can be solved in polynomial time for specific topologies. Also, efficient approximation algorithms have been proposed for both static and online traffic.

This section is organized as follows. We first consider the path topology where the problem can be solved in polynomial time. Then we review efficient approximation algorithms for the ring topology where the problem is already NP-hard, and also for more general topologies. Finally, we show how game theory can be useful to solve dynamic and online versions of the problem.

2.3.1 Path

Let the physical topology be the directed path $P_N$ with $N$ nodes labelled $1, 2, \ldots, N$, and $N - 1$ arcs $(i, i+1)$ for $1 \leq i < N$. Let also $TT_N = \{(i, j), \ 1 \leq i < j \leq N\}$ denote a transitive tournament, that is, the set of all requests from left to right.

For any set of requests $I \subseteq \lambda TT_N$, where $\lambda$ is a positive integer, the problem of minimizing the number of LTEs on $P_N$ can be solved optimally in polynomial time using a greedy algorithm. To prove that, it is sufficient to observe that the number of LTEs needed at node $i$ of $P_N$ is equal to $\max\{d^-_I(i), d^+_I(i)\}$, where $d^-_I(i)$ (or $d^+_I(i)$) denote the indegree (or outdegree) of node $i$ in $I$, that is, the number of requests $\{u, i\}$ with $u < i$ (or $\{i, v\}$ with $i < v$). We obtain Proposition 2.1, and in Corollary 2.1 we give the exact number of LTEs when $I = TT_N$.

**Proposition 2.1 (Bermond et al. [4]).**

$A(P_N, I, 1) = \sum_{i=0}^{N-1} \max\{d^-_I(i), d^+_I(i)\}$.

**Corollary 2.1 (Bermond et al. [4]).**

$A(P_N, TT_N, 1) = \frac{3N^2 - 2N - \varepsilon}{4}$, where $\varepsilon = N \mod 2$.

When the physical topology is a bidirectional path, it is necessary to be precise about how LTEs can be used. In particular, one has to be precise about whether it is possible to share a LTE between requests $(u, i)$ and $(i, v)$ with $u, v < i$, that is, a left-to-right request ending at $i$ and a right-to-left request starting from $i$, or not. If it is not possible, then the problem can be decomposed into two subproblems on a directed path that will be solved independently. But when such sharing is allowed, the problem has not been addressed in the literature and it is conjectured to be NP-complete.
2.3.2 Ring

The problem of minimizing the number of LTEs in optical networks was introduced in [69] for the unidirectional ring topology. It is proved to be NP-hard independently in [58] and [84]. The NP-hardness proofs also apply to bidirectional rings, even when the routing of connection requests is given. An algorithm with approximation ratio of \( \frac{3}{2} \) was presented in [52] for unidirectional and bidirectional rings with given routing. This algorithm has a first step (called the preprocessing step) that finds cycles in the instance and colors each cycle with a unique color. The remaining requests are then merged to form chains. This algorithm can also be adapted to the case where also the routing has to be determined, with the same approximation ratio [52].

This technique was improved in [103], showing that if the preprocessing phase tries to remove short cycles first, then an approximation ratio of \( \frac{10}{7} + \varepsilon \) can be achieved. This is improved to \( \frac{10}{7} \) in [59] using the same technique with a more detailed analysis.

In [13], we give exact algorithms for the all-to-all set of requests on uni- and bidirectional rings. Surprisingly, these results are obtained using a partition of the set of requests into cycles of lengths 3 and 4.

In [53] and [60] a variant of this problem is considered. In this variant, a path can be broken into segments and each segment can be colored using a different wavelength. Obviously this might incur an additional cost in terms of LTEs, but it allows to reduce significantly the number of wavelength used.

2.3.3 General Topology

In [52] an approximation algorithm was presented for general networks. The algorithm has a preprocessing phase where cycles of length at most \( l \) are included in the solution; this algorithm was shown to have performance guarantee of \( \text{OPT} + \frac{1}{2} (1 + \varepsilon)N, 0 < \varepsilon \leq \frac{1}{l+2} \), where \( \text{OPT} \) is the cost of an optimal solution and \( N \) is the number of connection requests for any given odd \( l \). A special case of this algorithm is when there is no preprocesing (i.e., \( l = 1 \)). The analysis reduces in this case to \( \text{OPT} + \frac{2}{3}N \). The dominant part in the running time of the algorithm is the preprocessing phase, which is exponential in \( l \).

In [65] we improve the analysis of the algorithm of [52] and prove a performance of \( \text{OPT} + \frac{1}{2} (1 + \varepsilon)N, \frac{1}{2l+3} \leq \varepsilon \leq \frac{1}{2(l+2)} \). Specifically, we show that the algorithm guarantees an upper bound of \( \text{OPT} + \frac{1}{2} (1 + \varepsilon)N \) for \( \varepsilon \leq \frac{1}{2(l+2)} \), and we demonstrate a family of instances for which the performance of the algorithm is \( \text{OPT} + \frac{1}{2} (1 + \varepsilon)N \) for \( \varepsilon \geq \frac{1}{2l+3} \).

Our analysis sheds more light on the structure and properties of the algorithm by closely examining the structural relation between the solution found by the algorithm and an optimal solution for any given instance of the problem. As the running
time of the algorithm is exponential in $l$, our result implies an improvement in the analysis of the running time of the algorithm. For any given $\varepsilon > 0$, the exponent of the running time needed to guarantee the approximation ratio $(3 + \varepsilon)/2$ is reduced by a factor of $3/2$. In addition, in the development of our bounds we address a purely combinatorial problem, which is of interest by itself.

We also improve the analysis for the special case where there is no preprocessing. In [64] we develop a new technique for the analysis of the upper bound and prove a tight bound of $OPT + \frac{3}{2}N$ for the performance of this algorithm.

### 2.3.4 Online Traffic

In many applications the requests arrive at the network online, and we have to assign them wavelengths so as to minimize the switching cost. In more involved cases we have also to determine the actual routing for these requests. In these situations, once an assignment is made the system cannot change it, and the aim is to suggest a strategy that will optimally utilize the network resources. Such a study is thus of great importance in the operation of optical networks.

Formally, an online algorithm is said to be $c$-competitive if, for any sequence of inputs, the cost is at most $c$ times that of an optimal off-line algorithm (see [15]).

In [102] we present an online algorithm for the problem of minimizing the number of LTEs, and prove that its competitive ratio is $\frac{7}{4}$. We show that this result is the best possible in general. Moreover, we show that even for the ring topology network there is no online algorithm with competitive ratio better than $\frac{7}{4}$. We show that on the path topology the competitive ratio of the algorithm is $\frac{3}{2}$. This is the best possible for this topology. The lower bound on the ring topology does not hold when the ring is of bounded size. We analyze the triangle topology and show a tight bound of $\frac{5}{4}$ for it. The analysis of the upper bounds, as well as those for the lower bounds use all a variety of proof techniques, which are of interest on their own, and which might prove helpful in future research on the topic.

### 2.3.5 Price of Anarchy

Game Theory and the associated concept of Nash equilibria have recently emerged as a powerful tool for modeling and analyzing a lack of coordination in distributed environments. In this setting, each communication request is handled by an agent (or player) selfishly performing moves, i.e., changing her routing strategy in order to maximize her own benefit. A Nash equilibrium is a solution of the game in which no agent gains by unilaterally changing her routing strategy. If Nash equilibria are reached in a polynomial number of selfish moves, and finding an improving user move is a problem solvable in polynomial time, such a non-cooperative process naturally defines a distributed polynomial-time algorithm. However, due to the lack
of cooperation among the players, Nash equilibria are known not to always optimize the overall performance. Such a loss has been formalized by the so-called price of anarchy (or optimistic price of anarchy), defined as the ratio between the cost of the worst (or best) Nash equilibrium and the one of a centralized optimal solution. The notion of Nash equilibria goes back to [91]. For about a decade the use of game theory has gained a lot of attention in numerous computer science directions, in what is known today as algorithmic game theory (see [92, 100]). The notion of price of anarchy goes back to [82].

In [54] we consider non-cooperative games in all-optical networks where users share the cost of the LTE switches used for realizing given communication patterns. We show that the two fundamental cost sharing methods, Shapley and Egalitarian, induce polynomial converging games with price of anarchy at most $\frac{5}{3}$, regardless of the network topology. Such a bound is tight even for rings. Then, we show that if collusion of at most $k$ players is allowed, the Egalitarian method yields polynomially converging games with price of collusion between $\frac{3}{2}$ and $\frac{3}{2} + \frac{1}{k}$. This result is very interesting and quite surprising, as the best-known approximation ratio, that is $\frac{3}{2} + \varepsilon$, can be achieved in polynomial time by uncoordinated evolutions of collusion games with coalitions of increasing size.

Moreover, with respect to the optimization of the optical spectrum, in [14] we investigate the problem in which a provider must determine suitable payment functions for non-cooperative agents wishing to communicate so as to induce routings in Nash equilibria using a low number of wavelengths. We assume three different information levels specifying the local knowledge that agents may exploit to compute their payments. Under complete information of all the agents and their routing requests, the network provider can compute prices where a Nash equilibrium is reached such that the assignment is the same as the one computed by a centralized algorithm. If the price to an agent is based only on the wavelengths used along connecting paths (minimal level) or along the edges (intermediate level), the most reasonable functions either do not admit equilibria or admit equilibria with the worst possible price of anarchy, that is, the ratio between the number of colors used by the worst Nash equilibrium and the one used by an optimal solution. However, by suitably restricting the network topology, a constant price of anarchy for chains and rings and a logarithmic one for trees have been obtained under the minimal and intermediate levels, respectively.

For more information, we refer to Chapter 9.

### 2.4 Minimizing the Number of Add/Drop Multiplexers

We will now concentrate on the case where the grooming factor is $g > 1$, for which we gave examples in Section 2.2. We first survey the main complexity and (in)approximability results. Then we see that for particular topologies and sets of connection requests, it is possible to obtain optimal constructions. We also consider the case where ADMs are placed a priori.
Let us first clarify the difference between single-hop and multi-hop (or bifurcation allowed) routing in this context. With single-hop routing, each request is routed through the same wavelength from its source to its destination. This is used for simple network topologies such as directed paths or rings, but not for general topologies where multi-hop routing is needed. When multi-hop routing is allowed, a request may be switched from one wavelength to another at intermediate nodes. This gives more flexibility for the traffic aggregation that is useful to optimize simultaneously the number of ADMs and wavelengths (see Section 2.5).

2.4.1 Complexity and Inapproximability Results

Determining the NP-completeness of the traffic grooming problem has been an open question for many years. It was first proved NP-complete on unidirectional rings in [21] using a reduction from the Bin Packing problem. Another proof was also mentioned in [112]. Later, the NP-completeness result has been refined.

More precisely, in [101] the traffic grooming problem is shown to be NP-complete in the strong sense for a given grooming factor \( g \geq 2 \), a network of directed path (or unidirectional ring) topology, a set of demands \( I \subseteq K_N \), single-hop routing, and an unbounded number of colors (wavelengths). It is also shown to be NP-complete for rings and for paths for any fixed value \( g \geq 2 \), and when the number of colors is bounded.

The traffic grooming problem has also been proved NP-complete and hard to approximate in star networks in [74]. These results have been extended in [62] where a complete characterization of the traffic grooming problem complexity in star networks is given by providing optimal polynomial-time algorithms for \( g \leq 2 \) and proving the intractability of the problem for any fixed \( g > 2 \).

The first inapproximability result for traffic grooming with fixed values of the grooming factor \( g \) has been obtained in [2], thus answering affirmatively the conjecture of [23]. More precisely, it has been proved that traffic grooming on a unidirectional ring for fixed \( g \geq 1 \) and traffic grooming on a directed path for fixed \( g \geq 2 \) are APX-complete. That is, there is no polynomial-time approximation scheme (PTAS) with constant approximation factor for these problems, unless P = NP. Both results rely on the fact that finding the maximum number of edge-disjoint triangles in a graph (and more generally cycles of length \( 2g + 1 \) in a graph of girth \( 2g + 1 \)) is APX-complete.

In particular, this implies that the traffic grooming problem is NP-complete in rings for fixed \( g \geq 1 \) and in paths for fixed \( g \geq 2 \) for an unbounded number of wavelengths, extending in this way the results of [101].
2.4.2 Approximation Results

The first approximation algorithm for the traffic grooming problem has been designed for the ring topology [71]. It is based on a greedy partition of the set of connection requests into trees of width at most $g$ and has approximation ratio $\sqrt{g}$.

In [63] we present an approximation algorithm for the problem of minimizing the number of ADMs on a general network in the case where grooming is allowed. For every value of the grooming factor $g$ the running time of the algorithm is polynomial in the input size. The approximation ratio of this algorithm for a wide variety of network topologies – including the ring topology – is shown to be $2\ln g + o(\ln g)$. In [62] the approximation ratio of the algorithm is shown to be $2\ln(\delta \cdot g) + o(\ln(\delta \cdot g))$ for any undirected tree having fixed node degree bound $\delta$, and $2\ln g + o(\ln g)$ for unbounded degree directed trees.

As we have seen above, for general grooming factor $g$ the best approximation algorithm [63] for the traffic grooming on a ring achieves an approximation factor of $O(\log g)$, but its running time is exponential in $g$ (that is, $N^g$). However, in practical applications such as SONET/SDH WDM rings, which are widely used as backbone optical networks [57], the grooming factor is equal to 3 or 4, typically when four 655 Mbit/s streams are aggregated into one 2.5 Gbit/s wavelength.

It is also important to find good approximation algorithms with running time polynomial in both $N$ and $g$. Such approximation algorithm has been proposed in [2], where $g$ is considered as part of the input. To the best of our knowledge, this is the first polynomial-time approximation algorithm for the traffic grooming problem with an approximation ratio which does not depend on $g$.

**Theorem 2.1 (Amini et al. [2]).** There exists a polynomial-time approximation algorithm that approximates the traffic grooming problem on a ring within a factor of $O(N^{1/3}\log^2 N)$ for any grooming factor $g \geq 1$.

**Theorem 2.2 (Amini et al. [2]).** There exists a polynomial-time approximation algorithm that approximates the traffic grooming problem on a path within a factor of $O(N^{1/3}\log^2 N)$ for any grooming factor $g \geq 2$.

Although the performance of this algorithm seems not to be very good at first sight, in fact it is conjectured in [2] that for the general instance of the problem it is not possible to get rid of a factor $n^\delta$ for some constant $\delta > 0$.

Finally, in [2] it is shown that the general scheme of the algorithm yields an $O(\log^2 N)$-approximation if the request graph excludes a fixed graph as minor, for example, if $R$ is planar or of bounded genus. The main theoretical contribution of this algorithm is to relate the traffic grooming problem to the dense $k$-subgraph problem [61] and the degree constrained subgraph problem [1].
2.4.3 Specific Constructions

For specific grooming factors, sets of requests and topologies, it is possible to give optimal constructions (assignment of requests to wavelengths that minimizes the number of ADMs). This is typically the case with all-to-all unitary traffic (one unitary request between each pair of nodes) where optimal constructions have been obtained on simple topologies for a specific grooming factor.

In unidirectional rings, all requests are routed clockwise. Therefore, it is possible to route requests \((i,j)\) and \((j,i)\) on the same wavelength at the cost of two ADMs and using \(\frac{1}{g}\) of available bandwidth all along the ring. When the set of requests is symmetric, this is shown to be optimal [11]. Furthermore, in this case, the set of requests can be modeled by an undirected graph, each edge corresponding to a circle, and a subgraph \(B\) with \(g\) edges corresponds to a valid assignment of \(g\) circles to a wavelength. The number of nodes of \(B\) gives the number of ADMs to use on the corresponding wavelength. Therefore, the traffic grooming problem on a unidirectional ring with symmetric traffic and grooming factor \(g\) can be modeled as the following partition problem.

Definition 2.1 (Traffic Grooming in Unidirectional Ring with Symmetric Traffic).

Input: \(N\) nodes unidirectional cycle \(C_N\), grooming factor \(g\), and set of symmetric requests modeled by graph \(I\).

Output: Partition of \(I\) into subgraphs \(B_w\), \(1 \leq w \leq W\), such that \(|B_w| \leq g\).

Objective: Minimize \(\sum_{w=1}^{W} |V(B_w)|\), and the optimum is denoted \(A(C_N, I, g)\).

This problem is in general NP-complete. However, for the all-to-all unitary set of traffic requests, \(I = K_N\), the complexity of the problem is unknown so far. Indeed, it is clearly a difficult combinatorial problem. Using tools of Design Theory [47], optimal constructions have been obtained for grooming factor \(g = 3\) [5], \(g = 4\) [11, 73], \(g = 5\) [7], \(g = 6\) [6], \(g = 7\) [48], and \(g \geq N(N-1)/6\) [11]. It has also been solved for practical values of \(N\) and \(g\) [9], that is, \(N \leq 16\) and \(g = 3, 4, 12, 16, 48, 64\).

When the physical topology is a directed path, the problem has only been solved for grooming factor \(g = 2\), with all requests from left to right (transitive tournament, \(TT_N\)) [4]. As for traffic grooming on a unidirectional ring, the problem can be modeled as a graph partition problem. The main difficulty here is that the number of connections in each subgraph is subject to high variation since, for example, all requests \((i,i+1)\) may fit in the same subgraph (see [8] for the maximum value for any \(g \geq 1\)), and no suitable tools from graph or design theory have been developed so far. A formal definition of the problem for any valid set of connection requests is given in Definition 2.2, where load \((B_w,e)\) denotes the number of requests of \(B_w\) routed in the path through edge \(e\).

Definition 2.2 (Traffic Grooming in Directed Path).

Input: \(N\) nodes directed path \(P_N\), grooming factor \(g\), and set of requests \(I\).

Output: Partition of \(I\) into subgraphs \(B_w\), \(1 \leq w \leq W\), such that \(\text{load}(B_w,e) \leq g\) for all \(e \in P_N\).

Objective: Minimize \(\sum_{w=1}^{W} |V(B_w)|\), and the optimum is denoted \(A(P_N, I, g)\).
Table 2.1 Congruence classes of $N$ for some $g$ for which optimal constructions are given

<table>
<thead>
<tr>
<th>$k$</th>
<th>$g$</th>
<th>$N$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>All values</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>$N \equiv 1, 5 \mod 12$</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
<td>$N \equiv 1, 7 \mod 24$</td>
</tr>
<tr>
<td>4</td>
<td>10</td>
<td>$N \equiv 1, 9 \mod 40$</td>
</tr>
<tr>
<td>5</td>
<td>15</td>
<td>$N \equiv 1, 9 \mod 30$</td>
</tr>
<tr>
<td>6</td>
<td>21</td>
<td>$N \equiv 1, 13 \mod 84$</td>
</tr>
<tr>
<td>7</td>
<td>28</td>
<td>$N \equiv 1, 15 \mod 112$</td>
</tr>
<tr>
<td>8</td>
<td>36</td>
<td>$N \equiv 1, 17 \mod 144$</td>
</tr>
</tbody>
</table>

Finally, when the physical topology is a bidirectional ring, the routing of the requests has to be taken into account since shortest path routing is not always optimal in general. However, it has been proved in [12] that symmetric shortest path routing allows us to obtain optimal solutions on bidirectional rings with all-to-all unitary traffic. The main results in this case are the following: optimal construction for the particular case $g = 1$ [13]; optimal construction when $g = 4, 8$ [49, 50]; optimal construction when $g = 3$ and $N \equiv 1, 5 \mod 12$ [12] and when $g = k(k + 1)/2$ for some congruence classes of $N$ summarized in Table 2.1; and construction with approximation factor $12/11$ when $g = 2$ [12].

2.4.4 A Priori Placement of the Equipment

In this section we study traffic grooming in unidirectional rings considering a wider range of requests than, for example, a complete graph. The idea is to place the ADMs in the nodes with limited knowledge of the graph of requests, for instance, knowledge of only its maximum degree. This model helps the network designer to take into account small traffic variations when deciding where to install ADMs, since in many situations one cannot expect to add or remove equipment at the nodes when the requests vary.

Namely, we consider the problem of placing the minimum number of ADMs in the nodes of a unidirectional ring in such a way that the network could support any request graph with maximum degree bounded by a constant $\Delta$. Note that using this approach, as long as the degree of each node does not exceed $\Delta$, the network can support a wide range of traffic demands without reconfiguring the equipment placed at the nodes. The problem can be formally stated as follows.

Definition 2.3 (Traffic Grooming in Unidirectional Rings with Bounded-Degree Symmetric Request Digraph).
Table 2.2 Values of $M(g, \Delta)$ found in [90]. The case $g = 4$ and $\Delta = 3$ is a conjectured value

<table>
<thead>
<tr>
<th>$g \setminus \Delta$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>...</th>
<th>$\Delta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1, 2</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>...</td>
<td>$\Delta$</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>≥ 3</td>
<td>≥ 4</td>
<td>≥ 4</td>
<td>...</td>
<td>≥ $\frac{2 \Delta}{3}$</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>2</td>
<td>2??</td>
<td>≥ 3</td>
<td>≥ 4</td>
<td>≥ 4</td>
<td>...</td>
<td>≥ $\frac{3 \Delta}{4}$</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>≥ 3</td>
<td>≥ 3</td>
<td>≥ 4</td>
<td>...</td>
<td>≥ $\frac{4 \Delta}{5}$</td>
</tr>
<tr>
<td>$g \geq 5$</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>3</td>
<td>4</td>
<td>...</td>
<td>$\geq \frac{g+1 \Delta}{g-2}$</td>
</tr>
</tbody>
</table>

Input: $N$ nodes unidirectional cycle $C_N$, grooming factor $g$, and a maximum degree $\Delta$.

Output: An assignment of $A(v)$ ADMs to each node $v \in V(C_N)$, in such a way that for any request graph $I$ (each edge represents a pair of symmetric requests) with maximum degree at most $\Delta$, there exists a partition of $I$ into subgraphs $B_\lambda$, $1 \leq \lambda \leq \Lambda$, such that:

(i) $|E(B_\lambda)| \leq g$ for all $\lambda$; and
(ii) each vertex $v \in V(C_N)$ appears in at most $A(v)$ subgraphs.

Objective: Minimize $\sum_{v \in V(C_N)} A(v)$, and the optimum is denoted $A(C_N, g, \Delta)$.

This problem has been studied in [90]. It solves the cases corresponding to $\Delta = 2$ (for all values of $g$) and $\Delta = 3$ (except for $g = 4$), and give upper and lower bounds for the general case. It also characterizes the function $A(C_N, g, \Delta)$, which turns out to be linear in $N$.

Lemma 2.1 ([90]). The function $A(C_N, g, \Delta)$ is of the form $A(C_N, g, \Delta) = MN - \alpha$, where $M$ and $\alpha$ are natural numbers depending only on $g$ and $\Delta$.

A summary of the results of [90] is given in Table 2.2, where $M(g, \Delta)$ is the smallest integer such that the inequality $A(C_N, g, \Delta) \leq M(g, \Delta)N$ holds for any $N \geq 1$.

2.5 Multilayer Traffic Grooming for General Networks

In previous sections we have discussed various aspects of traffic grooming for ring and tree networks. In this section we will discuss the case of more general network topologies, typically referred to as mesh networks. First we give an overview of different architectures of practical interest; then we give a survey of different graph models used with an ILP formulation and show examples of what can these models be used for.
2.5.1 Multilayer Mesh Networks

If there are multiple network layers “one over the other,” we refer to this structure as “Multilayer” network. It is also referred to as the vertical structure of networks, in contrast to the horizontal, where multiple domains are mutually interconnected. These network layers are not the ISO-OSI layers, where each layer defines some network functionality, but layers that can each provide certain connections or virtual connections and that can be established using the same or different network technologies.

Examples where the same network technology is used are the old FDM (Frequency Division Multiplexed) systems, different ATM (Asynchronous Transfer Mode) networks with two layers, namely VP and VC layers, and the MPLS (Multi-protocol Label Switching) networks where practically any number of LSPs (Label Switched Paths) can be established, where the lower-layer paths are considered as links in the upper-layer. In this case, the upper-layer paths share these lower-layer paths, i.e., they are encapsulated or embedded into these paths.

Examples where different technologies are used are

- PDH over SDH
- IP over PoS/MAPOS over SDH over WDM
- IP over ATM/MPLS over SDH over WDM
- IP over GFP over SDH over OTN over WDM
- IP over PPP over Ethernet over ATM-AAL5 over SDH over OTN ...

A multilayer network consists in general of interconnected multilayer and single-layer nodes. The single-layer nodes can be at any network layer, while multilayer nodes are those that are attached to two or more layers and/or perform the switching at two or more layers.

There are two general specifications of such multilayer architectures one referred to as GMPLS (Generalized Multi-protocol Label Switching) by the IETF [86] and the other ASTN (Automatic Switched Transport Network) by the ITU-T [76].

The IETF GMPLS framework [98] defines the following layers, this time according to the switching capability, i.e., a layer can be established by different networking technologies:

- PSC (Packet Switching Capable, e.g., IP)
- L2SC (Layer 2 Switching Capable, e.g., GbEth)
- TSC (TDM Switching Capable, e.g., SDH VC-4-4c)
- \( \lambda \)SC (Wavelength Switching Capable)
- WBSC (WaveBand Switching Capable)
- FSC (Fiber Switching Capable)

Typically not all these layers are represented in a network, but rather only two or three of them. Having multiple layers has both advantages and disadvantages. The advantages are that the services can access finer bandwidth granularity and some additional features of upper-layers only, i.e., for a small ratio of traffic only. The drawbacks are that some functionality is multiplied across layers and that the
complexity of operating multilayer networks is much higher than that of operating certain layers separately.

This layered vertical structure is valid for the data plane (DP), i.e., the network that carries the user information. However, for configuring and operating such a network we need a management and a control plane (MP and CP respectively).

If the DP layers of this vertical structure are run by different operators or providers, then they must communicate with each other to exchange information necessary for routing and other purposes. This vertical communication between MP and CP layers is referred to as Interconnection, and there are three defined Interconnection Models: (1) Overlay, (2) Augmented, and (3) Peer model [98].

The Overlay model is a client-server model where the upper (client) layer always adapts to the lower (server) layer. In the case of the Peer model, all necessary information is interchanged between the layers, and they may act together, e.g., in routing a demand. The Augmented (or hybrid) model is somewhere in between the Overlay and Peer models.

The DP layers in a node can be controlled either each by its own CP instance that communicates with other layers of that node, or by a single CP instance that controls all the DP layers of that node.

The latter case is feasible only if all the DP layers are run by a single operator or provider, since there is no need for communication interfaces between the layers. Therefore, a single unified integrated CP can be used for all the layers instead of the interconnection, the so-called Integrated Model. The forwarding units of all the layers of the data plane are connected to a single control plane unit.

Similarly, if such a multilayer network has layers or some parts of certain layers built of interconnected elements of a unique networking technology, or, rather switching capability, then the set of these elements is defined by the CCAMP WG of IETF as a Region. A network having multiple different regions is referred to as a Multi-region network [93, 104].

2.5.2 On Grooming in Multilayer Mesh Networks

In switched multilayer transport networks (e.g., ASTN/GMPLS) the traffic demands have typically bandwidth of orders of magnitude lower than the capacity of wavelength links (\(\lambda\)-links). Therefore, it is not worth assigning exclusive end-to-end wavelength paths (\(\lambda\)-paths) to these demands, i.e., sub-\(\lambda\) granularity is required. Furthermore, the number of wavelengths per fiber is limited and costly. To increase the throughput of a network with a limited number of wavelengths per fiber, traffic grooming capability is required in certain nodes.

Here we assume two layers only, i.e., a Wavelength Routing Dense Wavelength Division Multiplexing (WR-DWDM) Network and one layer built over it. In the WR-DWDM layer, a \(\lambda\)-path connects two physically adjacent or distant nodes. These two physical nodes will seem adjacent for the upper layer built over it. More generally, we can consider this two-layer approach as two layers of a 4–6 layer GM-
PLS/ASTN architecture [98]. However, not only is the framing and layering structure of interest, but the control plane proposed in the GMPLS/ASTN framework is as well.

This upper layer is an “electronic” one, i.e., it can perform multiplexing different traffic streams into a single $\lambda$-path via simultaneous time and space switching. Similarly, it can demultiplex different traffic streams of a single $\lambda$-path. Furthermore, it can perform re-multiplexing as well: Some of the demultiplexed demands can be again multiplexed into some other $\lambda$-paths and handled together along them. This is often referred to as (traffic) grooming [27]. The electronic layer is required for multiplexing packets coming from different ports (asynchronous time division multiplexing).

This upper electronic layer can be a classical or “next generation” SDH/SONET, MPLS, ATM, GbE, or 10 GbE, or it can be based on any other technology. However, in all cases the network carries mostly IP traffic. The only requirement is that it must be identical for all traffic streams that have to be demultiplexed, and then multiplexed again, since we cannot multiplex, e.g., ATM cells with Ethernet frames directly.

### 2.5.3 Graph Models for Multilayer Grooming

Optical metro and particularly core networks consist of multiple layers, where multiple different networking technologies are stacked one over the other. For simplicity, here we assume two layers only, e.g., an IP/MPLS layer over an DWDM layer, both controlled jointly by either one vertically peer-interconnected or one vertically integrated GMPLS control plane.

To better utilize network resources, smaller, upper-layer traffic streams are multiplexed (“groomed”) into higher capacity wavelength paths in a distributed way throughout the network.

In this section we give an overview of known graph models as well as propose some new graph models that all allow both static and dynamic grooming while performing design, dimensioning, configuration, routing, multicasting, traffic engineering, and resilience functions.

#### 2.5.3.1 Grooming and Wavelength Assignment for Static Routing

The aim of the Grooming and Wavelength Assignment for Static Routing problem (or, for short, Static Grooming problem) is to find a static configuration of the virtual (logical) topology, and to assign the upper layer demands to this topology. It is assumed that the lower network topology, the number of wavelengths per link, the capacity of these links, and the traffic matrix is given.
The simplest case of static grooming is when the routing is given, and the routes of certain demands are to be bundled (groomed together) in certain parts of the network and assigned to a wavelength.

In [24, 31, 33], a simple model and various heuristic algorithms based on simulated annealing, threshold accepting, and tabu search, as well as a genetic algorithm, are proposed and evaluated. The idea of the model is that each part of a route along each link can be assigned to any wavelength if that wavelength has enough free capacity to accommodate the considered demand. The objective is to have as few groomings and wavelength conversions as possible. The elementary heuristic step is to try out different combinations of assigning a segment of a path to different wavelength links, where the improvements are accepted with higher probability.

The first model for static grooming where the routing was not given in advance but performed simultaneously with grooming and wavelength assignment was proposed in [32]. Later, a method based on ILP formulation for optimal configuration was proposed in [43], and due to complexity simple heuristic methods using the same graph model were proposed in [44].

The wavelength graph model proposed in [32] is as follows. For each fiber link \( l = (u, v) \) with \( \Lambda \) wavelengths from \( u \) to \( v \) we create \( \Lambda \) arcs, one per wavelength, from vertex \( u_{l, \lambda} \) to vertex \( v_{l, \lambda}, 1 \leq \lambda \leq \Lambda \). Thus, node \( u \) with \( L_{in} \) incoming links and \( L_{out} \) outgoing links is associated with vertices \( u_{in, \lambda} \) and \( u_{out, \lambda}, 1 \leq l_{in} \leq L_{in} \) and \( 1 \leq l_{out} \leq L_{out} \), and a bipartite digraph from vertices \( \{ u_{in, \lambda} \} \) to vertices \( \{ u_{out, \lambda} \} \) modeled possible interconnections in network node \( u \). This bipartite digraph will be complete if it is possible to switch from any wavelength to any other.

The ILP formulation [43] uses the proposed graph model, and finds the minimal cost multi-commodity flow over the graph according to the traffic matrix and the costs assigned to the edges of the graph. However, the ILP can be solved optimally for very small instances only.

Heuristics based on the decomposition into as many shortest path searches as nonzero elements in the traffic matrix were proposed in [44]. Here, empirical weighting of edges has been also proposed to improve the quality of results. In contrast to the ILP that gives exact globally optimal results (for very small network instances), this approach is an approximation only. It is however easily scalable to very large networks, since it is based on Dijkstra’s algorithm.

In [110] a heuristic method based on decomposition and iterations has been proposed that also contains elements of simulated annealing and tabu search. The idea was that an element of a traffic matrix is a demand that goes from node \( a \) to node \( c \); however, instead of setting up an end-to-end wavelength path we can use two shorter lightpaths via an intermediate node \( b \). Then it corresponds to a new traffic matrix, where elements \( a \) to \( b \) and \( b \) to \( c \) are increased by the bandwidth of demand \( a \)–\( c \) while this \( a \)–\( c \) entry is decreased by its bandwidth (typically to 0). In this case a simpler graph model was used [22] that originally did not support grooming but only wavelength routing and assignment in a single-layer network; however, grooming was handled through the traffic matrix transformations.

The use of Integer Linear Programming ensures that the solution is the global optimum in terms of the given objective function. However, as the problem to be
solved becomes more complex, and as the network size increases, ILP can become intractable (in particular for NP-hard problems). Still, it is worth using it as a reference, at least for smaller networks. As computing capacity grows, particularly due to the parallelism of supercomputers, GRIDs, and clusters this will also become a viable solution.

As already mentioned, in [43] an ILP formulation for the wavelength graph has been given. In [25, 26] the formulation has been extended for undirected graphs as well, with protection either at the upper or at the lower layer.

### 2.5.3.2 Network Dimensioning and Grooming Node Placement

For a two-layer network, both the layers and the interconnection points between the two layers must be dimensioned properly. However, due to the interactions of the layers, all three must be dimensioned simultaneously, leading to high complexity.

In a network it is not necessary to equip all the nodes with grooming capability. Furthermore, since the O/E (Opto-Electronic) and E/O (Electro-Optical) converters are very expensive, their numbers should be properly determined to reduce costs while maintaining proper operation of the network. In [94] three methods are proposed for deciding which nodes should perform grooming, and to dimension their grooming capacity. The three methods are a greedy approach, a vertex-cover-based approach, and a heuristic approach that sorts the nodes according to their eligibility for accommodating grooming capability. The three methods have similar performance. In all cases the wavelength graph has been used.

In [96] a simulation-based iterative heuristic method has been proposed. Its idea is that simulations are run for infinite grooming capability in all nodes, and statistics (probability density functions, or pdfs) of the resource usage are compiled. Based on these pdfs it is decided in which nodes to keep the grooming capability and how much to reduce it. Then simulations are repeated and the whole process continued iteratively.

In [95] the optimization objective was extended to optimise not only the grooming capability, but simultaneously the number of wavelengths to be used per fiber as well.

### 2.5.3.3 Grooming for Dynamic Routing

“Grooming for dynamic routing” or “dynamic grooming” means, that in an operational network the new demands arrive while the demands already routed get terminated sooner or later, i.e., the network changes dynamically. In contrast to static grooming this is a less complex problem, since a single demand has to be routed at a time and groomed together with some existing demands; however, it is hard to say what is the globally optimal long-term strategy.

Here we discuss some related papers.
In [109] the information multi-domain multilayer (MD-ML) influence of delay of advertisements and inaccuracies due to the topology and link state aggregation is studied in an MD-ML network. The wavelength graph model has been used; however, this information is available only within the domains. Over domain boundaries a simplified aggregated graph is advertised.

In [38, 39] the advantages and drawbacks are investigated of having both layers switched according to user demands compared to the case where the WDM system is fixed, and only rarely reconfigured, while over this virtual topology the demands are dynamically routed. Here, an enhanced version of the wavelength graph is used that we refer to as the Grooming Graph or the Fragment Graph, where a wavelength path can be cut into two or more shorter pieces and two or more shorter wavelength paths can be concatenated into a longer one to reduce the load of the electronic layer.

Finally, [79] gives an overview of routing demands of different traffic parameters (e.g., very different bandwidths) over multilayer multi-domain networks.

2.5.3.4 VPN, oVPN, VP\(\lambda\)N and VON, oVON, VO\(\lambda\)N

Virtual Private Networks (VPNs), as well as Virtual Overlay Networks (VONs), are virtual networks set over real physical networks by separating a part of physical resources, e.g., link and switching capacities. We will refer to these jointly as VN (Virtual Networks). When multilayer networks are considered, two main options can be differentiated: First, when the virtual topology provided by the lower layer is shared among the VPNs or VONs of the upper layer; second, when the VN is the virtual topology, i.e., the wavelength paths are the links of the VN.

In [85] multi-fiber WDM networks are considered. In this paper full wavelength conversion capability is assumed in all nodes; therefore, no wavelength continuity constraint has to be obeyed, but only as many parallel links as the product of the number of existing fibers and wavelengths. Heuristics based on decomposition and Suurballe’s shortest pair of paths algorithms (cf. Section 1.5.2.1) are used to determine the best failure-resistant VPNs either demand-by-demand or VPN-by-VPN.

In [28, 87] open VPNs (oVPNs) are optimized by using ILPs while obeying the wavelength continuity constraint. ILP formulations for the cases without and with protection are given. For the case with protection two sub-cases are defined: One with external protection, where the network provider is supposed to protect the VPN, the other with internal protection, when the VPN is configured in such a way that if any link or node fails, the resources of the VPN are used for protection. Finally, in [88] more physical limitations are considered for setting up wavelength paths.

2.5.3.5 Grooming for Multicast Traffic

Services like TV or video distribution can be more efficiently provided using point-to-multipoint tree structures rather than many point-to-point connections. These ser-
services have become increasingly more popular, and the bandwidth used by these services has also grown, i.e., unlike standard definition digital video, high-definition video is already streamed.

If not a single channel, but rather a bundle of programs is streamed simultaneously, this bandwidth may achieve or even exhaust the capacity of a single wavelength channel. Therefore, performing the multicast at the optical layer via a splitter can be a much cheaper solution than loading the electronic layer with all the multicasting.

In [107] multicast trees are obtained by ILP. Breadth and depth constraints are obeyed, and it has been evaluated how many ports and how many wavelengths (resources in general) are needed for electronic and optical signal branching and how many for unicast as a reference.

The wavelength graph model has been used again; however, it had to be modified to allow branching of the optical signal, which was not allowed for unicast demands.

In [97] methods for periodical reconfiguration of multicast trees has been proposed for two-layer grooming-capable networks. Multicast trees (light trees) change dynamically in time due to the changing of multicast endpoints, which causes degradation of the tree. A significant amount of network resources can be saved by regular reconfiguration. The benefit of reconfiguration is investigated for different routing algorithms and reconfiguration periods.

In [45] various restoration mechanisms for multicast trees are considered.

### 2.5.3.6 Grooming and Resilience

In two-layer grooming-capable networks the demands can be routed over either the upper or the lower layers, or even using both layers. The same holds for routing the protection paths of these demands. For dedicated protection only an SRG (Shared Risk Group) disjoint path is to be sought; however, for the case of shared path protection this becomes more complex. Namely, not only the capacity is shared, but also are the O/E and E/O conversion ports as well as the wavelength paths.

In [25, 26] an ILP formulation for different Dedicated Protection Schemes is presented, while in [68] a decomposition-based heuristic method has been proposed for the same purpose. In [66] different methods based on running Dijkstra’s algorithm twice or Suurballe’s algorithm for static grooming are presented (cf. Section 1.5.2.1). In [67] the difference is that dynamic grooming is assumed, i.e., demands arrive one by one and are both routed and protected instantly.

In [34] shared protection is proposed and fairness issues in terms of dependence on bandwidth and distances are investigated.

In [41, 42] a new version of the wavelength graph model has been introduced that allows not only setting up and tearing down lightpaths, but also fragmenting and defragmenting them. The idea is that if there are no free wavelength paths in a node, then an existing wavelength path can be cut (“fragmented”) and the new demand is added or dropped at that point. If there are two consequent wavelength paths carrying the same demand or demands, these can be concatenated, i.e., “defragmented.”
Therefore we refer to this model as “Fragment Graph.” Here, the routing of working and shared protection paths are considered simultaneously.

In contrast to the previous papers in [78], an Ethernet over WDM overlay is considered, where we compare different configurations of the wavelength path system of the WDM layer and optimally set up MSTP (Multiple Spanning Tree Protocol) trees of the Ethernet layer.

All the methods discussed in this subsection use the wavelength graph model except the last one, which assumes an overlay model, so a simpler graph is sufficient.

2.5.3.7 Traffic Engineering for Traffic Grooming

The simplest definition of Traffic Engineering (TE) is to “put the traffic where enough resources are available.” It can be considered as an improved adaptive routing. The adaptivity can be achieved in two ways. First, by setting edge weights in our graph to avoid congestions and higher blocking before they occur (“a priori”). Second, by applying wavelength path fragmentation and defragmentation as already explained in Subsection 2.5.3.6 to resolve existing congestions for newly arriving demands (“a posteriori”). Here we give a short overview of MLTE-(Multilayer Traffic Engineering)-related papers.

A general overview of TE in GMPLS controlled multilayer networks is presented in [111].

Several adaptive edge metrics (weights) for MLTE have been proposed and compared in [99], using a simpler graph model than in [80]. Then, adaptive fragmentation and defragmentation of wavelength paths is proposed in [35–37] and compared to the case with no fragmentation or defragmentation and to the case with OXCs only (i.e., no grooming capability). Next, [29] gives an overview of achievements of Routing TE and resilience in Heterogeneous-GMPLS-controlled networks, while [77] presents experimental results from European testbeds.

Finally, we discuss three papers [30, 40, 83] that perform joint “a priori” and “a posteriori” Traffic Engineering. The idea is that although the fragment graph (FG) is being used for performing “a posteriori” TE, and the edge weights of the FG are as follows. Assuming that roughly no more than half of the demands can be terminated and O/E – E/O converted, the load should be balanced accordingly. i.e., if there are few demands routed over the network and therefore few wavelengths are used, the longer wavelength paths with less electronic processing (grooming) are made cheaper. However, if more wavelengths start to be used, but the capacity of these wavelengths is not well utilized the cost of grooming will decrease leading to shorter paths over more and shorter fragments.
2.5.3.8 Cross-layer Optimization: Considering Physical Impairments While Routing

Often, in networks it is not enough to consider the available resources, but it is also necessary to consider the impairments that affect the signal quality at the physical layer and cause increased Bit Error Rate for services. This is a kind of cross-layer optimization, where the services are optimized with physical layer constraints.

The first use of grooming to repair the impaired signal was presented in [88] where such VPNs were configured, where the signal quality was satisfactory since the physical impairments were considered. In [120] the results were extended for routing in general. [107, 119] present deeper results on the same topic, while [46] gives an overview of the problem, and proposes an additional method for improving the signal quality by increasing the power level of signals that have to go far while decreasing the power levels of signals that go to closer destinations in order to avoid the harmful effect of nonlinear distortions.

2.6 Conclusion

The objective of this chapter was to present an overview of traffic grooming in connection-oriented networks (mainly in WDM networks) and the wide variety of mathematical tools used to address this issue. Traffic grooming refers to techniques used for an efficient sharing of the bandwidth offered by, e.g., a wavelength, using Time Division Multiplexing. It is usually associated with the routing of the requests and the survivability issue in single or multiple failure scenarios. Furthermore, traffic requests might be uni- or multicast, the traffic pattern may evolve with time, and the network could be multilayer. Therefore, traffic grooming is only part of the concerns addressed when designing and optimizing a network. But even when restricted to simple physical topologies (unidirectional path or ring) where the routing is fixed and with small grooming factor, the traffic grooming problem is difficult to solve and to approximate. Also, when all aspects have to be taken into account (traffic grooming, routing, survivability, and so on), problems to solve are so difficult that exact solutions are usually no longer expected, and it is essential to develop efficient heuristic algorithms. Some of them were presented in Section 2.5. Chapter 3 presents the state-of-the-art regarding exact approaches for this problem.

In this research area, several important questions are still open and further research are needed. In particular, when optimizing only the number of ADMs in SONET/SDH networks, practical values of the grooming factor are 3 and 4, but this is repeated several times from the slower 55 Mbit/s streams to the current 10 Gbit/s wavelengths. So, it is important to develop efficient optimization tools for grooming factors 3 and 4, but also to consider hierarchical problems in which unitary requests are combined by four into streams that are themselves combined by four, and so on.

In the general context, where traffic grooming is associated with routing and survivability issues, existing heuristic algorithms provide upper bounds without guar-
antee on the quality of the solution. Furthermore, the size of practical problems is too huge for existing mathematical tools. Therefore, research effort has to be put into the development of new mathematical tools allowing us to address large instances and to obtain optimal or near-optimal solutions.

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