Interference Analysis

5.1 Introduction

Throughout time, people have and will continue to use communications at an ever-increasing pace in an interference environment [1]. In addition to the widespread use of satellite systems during the decades of the 1970s and the 1980s, we are now living through the mobile revolution. A large percentage of the communications needs can be carried out satisfactorily, even in a bad interference situation, and people are willing to show moderation. For example, hearing a distant cochannel repeater when your local repeater is not active, while annoying, is not “unacceptable interference.” Hearing adjacent channel splatter while carrying on a conversation on simplex or your local repeater, while affecting the quality of the conversation, is not truly unacceptable interference. If it makes communication completely impossible, then it should be considered interference, although it still may not necessarily be harmful or willful. Take note at this point that many of the noise sources to be defined here do not affect FM/PM type radio operation except to cause desensing of the radio, possibly masking the desired signal. This is the reason we strive to define and derive the qualitative measures by which we can design modern wireless system in an ever-increasing interference background.

Up to this point, we have examined and analyzed distortion mainly in the form of fading that is caused to information signals by the wireless channel for the types of wireless systems currently being used. In this and the following chapters, we shall analyze and study interference and include
additive effects. We shall define signal to interference ratio ($S/I$, or SIR) as a quality measure and relate probability of error to carrier-to-interference ratio ($C/I$, or CIR) or $S/I$. In general terms, however, interference is considered in this book as any distortion agent to the desired signal. With the expected increase in congestion of frequency spectrum by the use of satellites, mobile systems, and wireless local loops (WLLs) in conjunction with various frequency reuse mitigation techniques in order to allow usage of higher bands, the role played by interference is likely to increase in the future. In the first three chapters, we introduced the design parameters of wireless systems in general, discussed the basic characteristics of the channel, analyzed coding, and defined the quality measure for various modulation techniques as the wireless system operates in an interference environment. We also defined the interference environment and recognized that we could categorize in characteristics into two groups. One is referred to the additive types of interference, which include cochannel, adjacent channel, intersystem intermodulation, and intersymbol. The other is referred to the multiplicative type, which is mainly the effect of multipath reflections, diffraction, and dispersion of transmitted signals as they enter the receiver of wireless systems, especially mobile. The effects of this type of interference are analyzed in Chapter 4. In this chapter, we shall analyze and discuss the additive type of interference. We shall also point out the parameters that will be incorporated in Chapter 6 to develop realistic interference-reduction techniques.

5.2 Types of Interference

The interference signals in wireless communication systems can be placed in two categories for the purposes of this chapter: those caused by natural phenomena, which are not within our capability to eliminate, and those manmade signals that, by and large, can be attenuated or controlled. Our objective here is to define interference as a signal that affects communications, define its sources, and then point out those methods that can be used in the design of modern wireless systems, in order to have acceptable communications in this type of setting.

5.2.1 Cochannel Interference

Cochannel interference is defined as the interfering signal that has the same carrier frequency as the useful information signal. For analysis purposes, we utilize the conditional cochannel interference probability (CCIP) measure [2].
Blocking probability or CCCIP is defined as the probability that the undesired signal local mean power (LMP) exceeds the desired LMP by a protection ratio denoted as $\beta$. Amplitude fading in a multipath pico- or microcellular environment may follow different distributions depending on the area covered, presence or absence of a dominating strong component, and some other conditions. For example, the motion of people within a building causes Rician fading in LOS paths, while Rayleigh fading still dominates in non-LOS paths. The Rician distribution contains the Rayleigh distribution as a special case and simultaneously is well approximated by a Gaussian distribution.

The calculation of the CCIP in a Nakagami mobile environment is particularly important, as Nakagami fading is one of the most appropriate models in many mobile communication practical applications. Nakagami distribution (also called $m$-distribution) contains a set of other distributions for special cases and provides the optimum in analyzing data from outdoor and indoor environments.

The CCIP $P_c$ can be expressed as [2]:

$$P_c = \text{Prob} \left( \frac{s}{k} \sum_{i=1}^{k} I_i < \beta \right)$$  \hspace{1cm} (5.1)

where

- $s$ = the LMP of the desired signal;
- $I_i$ = the LMP of the $i$th interferer;
- $\beta$ = the protection ratio;
- $k$ = the number of interferers.

If $w = s - \beta \cdot \sum_{i=1}^{k} I_i$ then (5.1) can be written as follows

$$P_c = \text{Prob} \left( w > 0 \right)$$  \hspace{1cm} (5.2)

Considering log-normal PDFs for the $I_i$ and $s$, the following expressions are given
\[ p_i(y_i) = \frac{1}{\sqrt{2 \cdot \pi \cdot \sigma_i}} \exp\left(\frac{-(\ln y_i - m_i)^2}{2\sigma_i^2}\right), \quad y_i \geq 0 \quad (5.3) \]

where

\[ y_i = \text{the LMP of the } i\text{th interferer;} \]
\[ \sigma_i = \text{the standard deviation of the LMP of the } i\text{th interferer.} \]

\[ p_S(y) = \frac{1}{\sqrt{2 \cdot \pi \cdot \sigma_S}} \exp\left(\frac{-(\ln y - m_S)^2}{2\sigma_S^2}\right), \quad y \geq 0 \quad (5.4) \]

where

\[ y = \text{the LMP of the desired signal;} \]
\[ \sigma_S = \text{the standard deviation of the LMP of the desired signal.} \]

It can be proven that the PDF of the \( \beta I_i \) is given [2]:

\[ p_{\beta I_i}(y) = \frac{1}{\beta} p_{I_i}\left(\frac{y}{\beta}\right) \quad (5.5) \]

Let \( \Phi_W(r) \), \( \Phi_S(r) \), \( \Phi_{\beta I_i}(r) \) be the characteristic functions of the variables \( w \), \( s \), \( \beta I_i \), respectively. By taking into account that \( s \), \( I_i \) are statistically independent, the following can be written [2]:

\[ \Phi_W(r) = \Phi_S(r) \cdot \prod_{i=1}^{k} \Phi_{\beta I_i}(-r) \quad (5.6) \]

Using the definition of the characteristic function, (5.6) assumes the form

\[ \Phi_W(r) = \Phi_S(r) \cdot \prod_{i=0}^{\infty} \int_0^\infty \exp(-irx_i) \cdot f_{\beta I_i}(x_i) \, dx_i \quad (5.7) \]

where
\[ f_{\beta I_i}(x) = \frac{1}{\beta} \cdot f_{I_i}(x) = \frac{1}{\beta} \cdot \left( \frac{m_k}{\Omega_k} \right)^{m_k} \cdot \frac{x^{m_k-1}}{\beta^{m_k-1} \cdot \Gamma(m_k)} \cdot \exp \left( -\frac{m_k}{\Omega_k} \cdot \frac{x}{\beta} \right) \] (5.8)

where \( f_{\beta I_i}(x) \) could be the log-normal or other well-known PDFs as m-Nakagami

\[ m_k = \text{an arbitrary fading parameter;} \]
\[ \Omega_k = \text{the average power.} \]

\[ \Gamma(r) \equiv \int_{0}^{\infty} \rho^{r-1} e^{-\rho} \, d\rho \]

where

\[ \Gamma(x) = \text{the Gamma function} \]

Setting \( \ln x_i - \ln \beta = m_i + \sigma_i \cdot r_i \) in (5.5), from (5.6) and (5.7) and making certain simplifications, we obtain:

\[ \Phi_w(r) = \frac{k}{2} \cdot 2\pi \cdot \Phi_S(r) \cdot \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} \exp \left( -j\tau \left( \sum_{i=1}^{k} \beta \cdot e^{(m_i + \sigma_i \cdot r_i)} \right) \right) \]

\[ \cdot \exp \left( \frac{-\sum_{i=1}^{k} r_i^2}{2} \right) \, dr_1 \cdots dr_k \] (5.9)

The random variable \( r_i \), which represents the amplitude of the \( i \)-th cochannel interferer, follows log-normal of Nakagami distribution. All of the \( r_i \) are statistically independent with \( r_i \geq 0 \).

But, using (5.2) and by definition, we have

\[ P_C = \int_{-\infty}^{0} f_W(\tau) \, d\tau = \frac{1}{2\pi} \cdot \int_{-\infty}^{0} \int_{-\infty}^{\infty} \Phi_w(r) \cdot \exp(-j\tau r) \, dr \, d\tau \] (5.10)

Now, using (5.5) in (5.10) and taking into account that by definition
\[
\int_{-\infty}^{\infty} \Phi_S(r) \cdot \exp \left[ -fr \left( \tau + \sum_{i=1}^{k} \beta \cdot e^{(m_i + \sigma_i \cdot r_i)} \right) \right] \, dr = 2\pi f \left( \tau + \sum_{i=1}^{k} \beta \cdot e^{(m_i + \sigma_i \cdot r_i)^2} \right)
\]

Then, the expression for \( P_C \) when \( f(x) \) is the log-normal PDF of the desired signal, then the \( P_C \) can be written as

\[
P_C = \frac{1}{(2\pi)^{k/2}} \cdot \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} \exp \left( -\sum_{i=1}^{k} \frac{r_i^2}{2} \right)
\cdot F \left( \sum_{i=1}^{k} \beta \cdot e^{(m_i + \sigma_i \cdot r_i)} \right) \, dr_1 \cdots dr_k
\]

where \( F(x) \) is the CDF given by

\[
F(x) = G_{NORMAL} \left( \frac{\ln x - m_S}{2\sigma_S} \right)
\]

with \( G_{NORMAL} \) being the CDF of the normal distribution. Hence, the final form for the \( P_C \) is

\[
P_C = \frac{1}{(2\pi)^{k/2}} \cdot \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} \exp \left( -\sum_{i=1}^{k} \frac{r_i^2}{2} \right)
\cdot G_{NORMAL} \left( \frac{\ln \beta - m_S + \sum_{i=1}^{k} e^{(m_i + \sigma_i \cdot r_i)}}{\sigma_S} \right) \, dr_1 \cdots dr_k
\]

where
\( \beta = \) the protection ratio in natural units;

\( \gamma = \) the path loss propagation factor;

\( \sigma_i = \) the standard deviation of the LMP of the interferers in natural units;

\( \sigma_s = \) the standard deviation of the LMP of the desired signal in natural units.

The second part of (5.14) can be calculated using the following Gauss-Hermite formula

\[
\int_{-\infty}^{\infty} \exp \left[ -x^2 \right] \cdot g(x) \, dx = \sum_{i=0}^{\nu} \alpha_i \cdot g(x_i)
\]

(5.15)

where

\( \alpha_i, x_i = \) constants given by special tables;

\( \nu = \) a constant that denotes the accuracy at the \( \nu \)th decade digit.

With the Gauss-Hermite formula, we can control the error in desired levels, but in a real cellular mobile radio environment, the shadow-fading parameter \( \sigma \) has different values in different regions of the system area. In this case, our formula for the CCIP is modified to

\[
P_c = \frac{1}{(2 \cdot \pi)^{k/2}} \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} \exp \left[ -\frac{\sum_{i=1}^{k} r_i^2}{2} \right]
\]

\[
\cdot G \left[ \ln \left( \beta \cdot (3 \cdot n_g)^{(-\gamma/2)} \cdot \left( e^{\sigma_1 \cdot r_1} + e^{\sigma_2 \cdot r_2} + \cdots \right) \right) \right] \, dr_1 \cdots dr_k
\]

(5.16)

with \( \sigma_1, \sigma_2, \ldots, \sigma_k \) as the standard deviations of the logarithm of the LMPs of the \( k \) interferers.

Because

\[
(3 \cdot n_g)^{1/2} = \frac{D}{R}
\]

(5.17)
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with

\[ R = \text{the radius of the cell and;} \]
\[ D = \text{the distance from the first tier;} \]
\[ n_g = \text{the cluster size.} \]

\( P_C \) can be written as

\[
P_C = \frac{1}{(2 \cdot \pi)^{k/2}} \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} \exp \left[ -\sum_{i=1}^{k} \frac{r_i^2}{2} \right] \]

\[
\cdot F \left[ \ln \left( \beta \cdot \left( \frac{D}{R} \right)^{-\gamma} \cdot (e^{\sigma_1 \cdot r_1} + e^{\sigma_2 \cdot r_2} + \ldots ) \right) \right] \frac{\sigma_s}{\sigma_s} dr_1 \cdots dr_k \tag{5.18}
\]

Equation (5.18) gives a general form for the CCIP in terms of the critical (for the cellular system) cochannel interference reduction factor \( D/R \). This is very important for the system designer because there is a direct connection between CCIP and this factor. Hence, giving a desired value for \( P_c \) in (5.18) and using an approximate mathematical method to solve this equation, the factor \( D/R \) can be calculated for several shadow and path loss environments of the system.

Equation (5.18) is true as long as the cell size is fixed and the cochannel interference is thus independent from the transmitted power of each cell. But, in the case where cell size is not fixed, the distances from the first tier are not the same for all the \( k \) interferers and (5.18) must be modified to

\[
P_c = \frac{1}{(2 \cdot \pi)^{k/2}} \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} \exp \left[ -\sum_{i=1}^{k} \frac{r_i^2}{2} \right] \]

\[
\cdot F \left[ \ln \beta - m_s + \ln \left( \sum_{i=1}^{k} e^{m_i \cdot \left( \frac{D_i}{R_i} \right)^{-\gamma} \cdot (\sigma_i \cdot r_i) } \right) \right] \frac{\sigma_s}{\sigma_s} dr_1 \cdots dr_k \tag{5.19}
\]
with

\[ m_s = \text{the area mean power of the desired signal}; \]
\[ R = \text{the radius of the cell contained the desired transmission}; \]
\[ R_i = \text{the radius of the cell contained the } i\text{th interferer and}; \]
\[ D_i = \text{the distance of the } i\text{th interferer from this cell}. \]

Blocking probability should be kept below 2%. As for the transmission aspect, the aim is to provide good quality service for 90% of the time. The analysis so far resulted in a simple criterion of relating design parameters such as D/R with quality of service in an interference environment.

### 5.2.2 Adjacent Channel Interference

The adjacent channel interference can be classified as either *inband* or *out-of-band* interference. The term *inband* is applied when the center of the interfering signal bandwidth falls within the bandwidth of the desired signal. The term *out of band* is applied when the center of the interfering signal bandwidth falls outside the bandwidth of the desired signal.

In the mobile radio environment, the desired signal and the adjacent channel signal may be partially correlated with their fades. Then the probability exists that \( r_2 \geq \alpha r_1 \), where \( r_1 \) and \( r_2 \) are the two envelopes of the desired and the interfering signals, respectively. In that case, the probability can be obtained from the joint density function, assuming that \( E[r_1^2] = E[r_2^2] = 2\sigma^2 \) and that \( \alpha \) is a constant

\[
P(r_2 \geq \alpha r_1) = \int_0^{\infty} dr_1 \int_{\alpha r_1}^{\infty} p(r_1, r_2) \, dr_2
\]

\[
= \int_0^{\infty} dr_1 \int_{\alpha r_1}^{\infty} r_1 r_2 \exp \left[ -\frac{r_1^2 + r_2^2}{2\sigma^2(1 - \rho_r)} \right] I_0 \left[ \frac{r_1 r_2 \sqrt{\rho_r}}{\sigma^2 (1 - \rho_r)} \right] \, dr_2
\]

\[
= \frac{1}{2} + \frac{1}{2} \cdot \frac{1 - \alpha^2}{\sqrt{(1 + \alpha^2)^2 - 4\rho_r \alpha^2}} \tag{5.20}
\]

where
\( \rho_r = \) is the correlation coefficient between \( r_1 \) and \( r_2 \).

The probability density function \( p_r(y) \) of \( r = r_2/r_1 \) can be obtained as follows

\[
p_r(y) \bigg|_{y=\alpha} = -\frac{d}{d\alpha} P\left( \frac{r_2}{r_1} \geq \alpha \right)
\]

We determine the term \( R = \sqrt{Gr} \), where \( G \) is the power gain at the intermediate frequency filter output for the desired signal relative to the adjacent channel interferer. Then, we have

\[
p_{R^2}(x) = p_r(y) \frac{1}{2yG} \bigg|_{y=\sqrt{x/G}} = \frac{(1 - \rho_r) \left( 1 + \frac{x}{G} \right)}{G \left[ \left( 1 + \frac{x}{G} \right)^2 - 4 \rho_r \frac{x}{G} \right]^{3/2}}
\]

(5.22)

where \( \rho_r \) is given by the formula

\[
\rho_r(\Delta \omega, \tau) = \frac{\gamma^2 (\beta V \tau)}{1 + (\Delta \omega)^2 \Delta^2}
\]

and with \( \tau = 0 \), it is simplified in the following form

\[
\rho_r = \frac{1}{1 + (\Delta \omega)^2 \Delta^2}
\]

(5.23)

where the term \( \Delta \omega/2\pi \) is the difference in frequency between the desired signal and the interferer. The term \( \Delta \) is the time delay spread. The \( \rho_r \) decrease, which will vary in value depending on the different types of mobile environments proportionately as either \( \Delta \) or \( \Delta \omega \) increases. As \( \rho_r \) decreases, the adjacent channel interference also decreases. The same procedure used to find the cochannel baseband SNR can also be used to find the baseband SNR, due to an adjacent channel interferer in a fading environment, by substituting the PDF of (5.19) in place of the PDF in a Rayleigh fading environment.
As a final consideration, when adjacent channel interference is compared with cochannel interference at the same level of interfering power, the effects of the adjacent channel interference are always less.

5.2.3 Intermodulation Interference

Nonlinear system components, especially in analog signal transmission, cause spurious signals, which may play the role of interference in adjacent channels. When a nonlinear device (amplifier) is used simultaneously by a number of carriers, intermodulation products are generated, which cause distortion in the signals. The nonlinearities in such cases are of two types: amplitude nonlinearities and amplitude to phase conversions (AM/PM), by which the change in the envelope of multicarrier input causes a change in the output phase of each signal component. In many instances, especially when the nonlinear element operates below saturation level, the AM/PM effects dominate the instantaneous amplitude nonlinearity.

In this section, we shall follow a procedure similar to the one described in previous sections and try to relate quality of communication with design system parameters in an environment, which operates in relation to phase intermodulation interference. Both nonlinearities will be treated jointly and the AM/PM conversion is modeled as follows.

Assuming an input signal of the following form [3]:

\[ s_1(t) = Re(Ae^{j\omega_0 t}) \]  

is used as an input to a nonlinear device, then the output of the particular nonlinear device with AM/PM characteristics is given by

\[ s_0(t) = Re g(A) \cdot e^{j(\omega_0 t + f(A))} \]  

where \( g(A) \) and \( f(A) \) are the amplitude and phase functions, respectively. For the rest of the analysis, (5.25) will represent the reference model for the nonlinearities we are going to consider.

In order to facilitate calculations, it is customary to use the approximation suggested in [4], which is given here:

\[ g(\rho) e^{j f(\rho)} \approx \sum_{\ell=1}^{L} b_{\ell} J_1(a_{\ell} \rho) \]  

Intermodulation effects caused by this type of nonlinearity are important in multicarrier signals, which will be considered next.
Assume that the input signal to a nonlinear device of the type described earlier is given by:

$$s_1(t) = R_e \left[ \sum_{i=1}^{M-1} A_i e^{j(\omega_0 t + j\theta_i(t))} + (N_c(t) + jN_s(t)) e^{j(\omega_0 + \omega_m)t} \right]$$

(5.27)

or

$$s_1(t) = R_e \left[ \sum_{i=1}^{m-1} A_i e^{j(\omega_0 t + \theta_i(t))} + A_m(t) e^{j(\omega_0 t + \theta_m(t))} \right]$$

(5.28)

where

$$\theta_m(t) = \omega_m t + \tan^{-1} \frac{N_s(t)}{N_o(t)}$$

$$A_i = \text{constant}$$

(5.29)

$$A_m(t) = \sqrt{N_c^2(t) + N_s^2(t)}$$

$$\theta_i(t) = \text{phase input carrier}$$

If this input multicarrier signal goes through a nonlinear device of the type described earlier, the output is given by [1–5].

$$s_0(t) = R_e \left[ e^{j\omega_0 t} \sum_{j=1}^{\infty} e^{j \sum_{j=1}^{m-1} k_j \theta_j(t)} \cdot M(k_1, k_2, \ldots k_m) \cdot e^{jk_m \theta_m(t)} \right]$$

(5.30)

where

$$M(k_1, k_2, \ldots k_m) = \int_0^\infty \gamma \prod_{i=1}^m J_{k_i}(\gamma A_i) \, d\gamma \cdot \int_0^\infty \rho g(\rho) e^{j\gamma \rho} \cdot J_1(\gamma \rho) \, d\rho$$

(5.31)
In the absence of a noise signal at the input of the device, the output, \( s_o(t) \), consists of the angle-modulated carriers and intermodulation products, which also have properties of angle-modulated carriers. With the introduction of noise at the input, the output may be divided into two categories:

1. The original output components with modified complex amplitudes;
2. Additional intermodulation components caused by the introduction of noise.

\[
s_o(t) = s_j(t) + s_N(t) \tag{5.32}
\]

These two classes can be represented by \( s_j \) and \( s_N \), respectively. For the particular case of Gaussian noise whose rms power is \( R(0) \), this yields

\[
s_j(t) = R e^{j\omega_0 t} \sum_{k_1, k_2, \ldots k_{m-1} = -\infty}^{k_1+k_2+\ldots+k_{m-1} = 1} e^{j \sum_{j=1}^{m-1} k_j \vartheta_j(t)} \cdot M_j(k_1, k_2, \ldots k_{m-1}) \tag{5.33}
\]

where

\[
M(k_1, k_2, \ldots k_{m-1}) = \int_0^\infty \gamma \prod_{i=1}^{m-1} J_{k_i}(\gamma A_i) e^{-\frac{\gamma^2}{2}} R(\omega) \, d\gamma \tag{5.34}
\]

This output signal can further be categorized into three types:

1. The main carrier to be demodulated;
2. The intermodulation products and noise falling within the band of the receiver filter of this main carrier;
3. The other carriers, intermodulation products, and noise falling away from the main carrier, which can be filtered out.
Categories 1 and 2 are important in the process of demodulating the main carrier. Before it passes the demodulator, and if we further assume for simplicity that \( k_1 = 1 \) and all other \( k_i = 0 \), the output signal can be represented as follows

\[
s_o(t) = R e^{j(\omega_0 t + \theta_1(t))} M_o (1 + R(t) + jI(t)) \tag{5.35}
\]

where

\[
M_o = \left[ \prod_{i=2}^{m-1} J_o(\gamma A_i) \right] J_1(\gamma A_1) C(\gamma) \cdot \int_0^\infty \rho g(\rho) e^{j\rho} J_1(\gamma \rho) d\rho \tag{5.36}
\]

\[
C(\gamma) = \int_0^\infty J_o(\xi) p(\xi) d\xi \tag{5.37}
\]

\( p(\xi) \) = probability density function of the noise amplitude \( \xi \) usually Rayleigh distributed;

\( R(t) \) = Real part of the expression in (5.38);

\( I(t) \) = Imaginary part of the expression in (5.38).

Hence,

\[
I(t) = I_m M_0^{-1} [M(1, 0, \ldots, 0; t) - M_0] + I_M \left\{ M_0^{-1} \cdot \sum_{k_1, k_2, \ldots, k_M = -\infty}^{k_1 + k_2 + \ldots + k_m = 1} M(k_1, k_2, \ldots, k_m; t) \right\} \tag{5.38}
\]

\[
\cdot e^{j k_m \left( \alpha_m t + \tan^{-1} \left( \frac{N_j(t)}{N_i(t)} \right) \right)} \cdot e^{j(k_1 - 1) \theta_1(t) + j \sum_{i=2}^{m-1} k_i \theta_i(t)}
\]

where \( M(1, 0, \ldots, 0, t) \) and \( M(k_1, k_2, \ldots, k_M; t) \) are given by expressions similar to that given in 5.31 and 5.34 [4], the output signal \( s_o(t) \) is then passed through an ideal angle demodulator.
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The output of an ideal demodulator therefore based on (5.35) is given

\[ S_0^1 = \phi_1(t) + \tan^{-1} \frac{I(t)}{1 + R(t)} \] (5.39)

Because in normal situations, \( I(t) \) and \( R(t) \) are small, the (5.39) can be approximated by

\[ S_0^1 \approx \phi_1(t) + I(t) \] (5.40)

In order to determine the effect of \( I(t) \) on the desired angle modulated signal, we need to calculate and determine the power spectrum of \( I(t) \). The power spectrum of \( I(t) \) in (5.40) as a function of the frequency is given in [3].

Having determined the power spectral density of \( I \), we can then form the ratio of signal power to noise power in the specified frequency band, as we shall see in Section 5.3.1.

For example, if the case under consideration is frequency modulation with multichannel telephony signals, the ratio is given by [5]

\[
\text{NPR}(f) = \frac{S}{N} = \frac{P(f) \cdot f_r \cdot f_{rms}^2}{(1 - \epsilon) r^2 f_r^2 S_I(f)}
\] (5.41)

\[
S_f = f^2 S_I(f)
\] (5.42)

where

- \( \text{NPR} \) = noise power ratio as a function of frequency;
- \( S_I(f) \) = assumed constant over a telephone channel;
- \( f_{rms} \) = rms frequency deviation;
- \( \epsilon \) = ratio of minimum to maximum baseband frequencies;
- \( f_r \) = top-based frequency of wanted signal;
- \( P(f) \) = the pre-emphasis weighting factor;
- \( r^2 = [C/I]^{-1} \) carrier to interference ratio as a function of frequency.
If we deal with the digital carriers, the impairment is measured in terms of the bit error probability, $P_e$, as we shall see later. For special cases, 4-phase (phase shift keying modulation) this parameter is given by

$$P_e = \frac{1}{2} \text{erfc}\left(\sqrt{\gamma}\right)$$  \hspace{1cm} (5.43)

where it is assumed that the interference is a close approximation to Gaussian noise and that $\gamma$ is the $S/N$ at the filter output at the sampling instant. We see that we have been able to relate $S/N$ and error probability with crucial design parameters of the wireless systems under consideration.

### 5.2.4 Intersymbol Interference

For several types of digital modulation, the equivalent lowpass transmitted signal has the following form [4–8]:

$$s_m(t) = \sum_{n=0}^{\infty} I_n u(t - nT)$$  \hspace{1cm} (5.44)

where $I_n$ represents the discrete information bearing sequence of symbols and $u(t)$ represents a pulse that, for simplicity, is assumed to have a bandlimited frequency characteristic $U(f)$ (i.e., $U(f) = 0$ for $|f| > W$).

We assume that the channel frequency response $C(f)$ is also band-limited such as $C(f) = 0$ for $|f| > W$.

The received signal has the form

$$s_o(t) = \sum_{n=0}^{\infty} I_n h(t - nT) + n(t)$$  \hspace{1cm} (5.45)

where

$$h(t) \equiv \int_{-\infty}^{\infty} u(t) c(t - r) \, dr$$  \hspace{1cm} (5.46)

$n(t)$ = represents additive Gaussian noise

The received signal is usually first passed through a filter and then sampled at the rate of $1/T$ samples per second.
We denote the output of the receiving filter as

\[ y(t) = \sum_{n=0}^{\infty} I_n x(t - nT) + V(t) \]  

(5.47)

where \( V(t) \) is the response of the receiving filter to the noise \( n(t) \). Sampling \( y(t) \) at sampling instants \( T \) seconds apart, we should be able to obtain the transmitted information symbol. The sampling gives

\[ y(kT + \tau_0) \equiv y_k = \sum_{\kappa=0}^{\infty} I_n x(kT - nT + \tau_0) + V(kT + \tau_0) \]  

(5.48)

\[ y_k = \sum_{n=0}^{\infty} I_n x_{\kappa-n} + V_k \]  

(5.49)

where

\[ x_{\kappa-n} = x(kT - nT + \tau_0) \]

\[ V_k = V(kT + \tau_0) \]  

(5.50)

\[ y_k = x_0 I_\kappa + \sum_{n=0}^{\infty} I_n x_{\kappa-n} + V_\kappa \]

Because \( x_0 \) is a scaling factor, we can set it arbitrarily to unity and thus the previous equation becomes

\[ y_k = I_\kappa + \sum_{n=0}^{\infty} I_n x_{\kappa-n} + V_\kappa \]  

(5.51)

In (5.51), the first term is the transmitted information symbol at the \( k \)th sampling instant and the second term

\[ \sum_{n=0}^{\infty} I_n x_{\kappa-n} \]  

(5.52)

is the unwanted signal (intersymbol interference), which is the interference contribution of other symbols to the symbol under consideration. \( v_\kappa \) is the
contribution of the additive Gaussian noise. This unwanted interference, depending on the type of modulation used, could be viewed on an oscilloscope as an eye pattern, or as a two-dimensional scatter diagram, as shown in Figures 5.1 and 5.2 [6].

It will be shown next that in order to eliminate this interference, the received signal must pass through a filter, which is matched to the received pulse. That is, the frequency response of the receiving filter should be $H^*(f)$.

![Figure 5.1](image-url)

**Figure 5.1** Eye pattern for binary pulse amplitude modulation (PAM).

![Figure 5.2](image-url)

**Figure 5.2** Two-dimensional digital eye patterns: (a) transmitted eight-phase signal and (b) received signal samples at the output of demodulator.
$H^*(f)$ is the complex conjugate of the frequency characteristic $H(f)$ of the input pulse $h(t)$.

If for simplicity we assume $C(f) = 1$ for all $|f| \leq W$, then $x(t)$ shown in (5.47) can be given by

$$x(t) = \int_{-W}^{W} X(f) e^{j2\pi ft} df$$

(5.53)

where

$$X(f) = U(f) U^*(f) = |U(f)|^2$$

(5.54)

For no intersymbol interference to exist, it is necessary that

$$x(t = kT) = 1 \quad \text{for} \quad k = 0$$

$$x(t = kT) = 0 \quad \text{for} \quad k \neq 0$$

(5.55)

Because $x(t)$ is a bandlimited signal, use of the sampling theorem gives [6]:

$$x(t) = \sum_{n=-\infty}^{\infty} x\left(\frac{n}{2W}\right) \frac{\sin 2\pi W \left(t - \frac{n}{2W}\right)}{2\pi W \left(t - \frac{n}{2W}\right)}$$

(5.56)

where

$$x\left(\frac{n}{2W}\right) = \int_{-W}^{W} X(f) e^{j2\pi f \frac{n}{2W}} df$$

(5.57)

If, moreover, we assume:

$$T = \frac{1}{2W}$$

(5.58)

and the symbol rate is the Nyquist rate, (5.56) becomes:
\[ x(t) = \sum_{n=-\infty}^{\infty} x(nT) \sin \left( \frac{n\pi}{T} (t - nT) \right) \]  
\[ (5.59) \]

For zero intersymbol interference, it is required that all \( x(nT) \) terms be zero except \( x(0) \). The previous equation then becomes

\[ x(t) = \sin \left( \frac{\pi t}{T} \right) \frac{\pi t}{T} \]  
\[ (5.60) \]

Three major problems are raised with this type of pulse in addition to the conditions set in order to eliminate ISI by (5.55).

1. This type of pulse is not physically realizable.
2. The tails of \( x(t) \) decay as \( 1/t \), and a mistiming error in sampling results in an infinite series of ISI components.
3. There is absolutely no flexibility in the symbol rate, but it must be precisely defined and restricted \( T = 1/2W \).

In practical situations, it is not possible to satisfy all three conditions simultaneously. If we impose the condition that the symbol rate be \( 2W \) symbols per second and remove the constraint that there is zero ISI, we obtain a class of physically realizable pulses called partial response signals. The compensation for the interference is then obtained through equalization and/or various optimization techniques. By these optimization techniques, we seek to obtain optimal receiver filter parameters with which, in turn, we obtain the best estimate of the received symbols. This, in effect, results in minimizing interference. The concept contained in this paragraph will be the cornerstone of some of the various methodologies, which will be developed to combat interference.

As far as equalization is concerned, the main thrust of the procedure lies in the fact that we seek to design discrete-time linear receiver filters to eliminate or reduce ISI—see (5.52)—which have impulse responses of the form

\[ q_n = \sum_{j=-\infty}^{\infty} c_j f_{n-j} \]  
\[ (5.61) \]
where \( q_n \) is simply the convolution of \( c_n \) and \( f_n \). Moreover, \( c_n \) is the impulse response of the equalizer, and \( f_n \) is the impulse response of the filter. In such a case, the estimate of the \( k \)th symbol is given by

\[
\hat{I}_k = q_0 I_k + \sum_{n \neq k} I_n q_{k-n} + V_k
\]  

(5.62)

The first term represents the scaled version of the desired symbol, which can be normalized to unity. The second term is the ISI, and the third term represents the noise. A standard procedure that leads to acceptable filter designs is to find the tap weight coefficients \( c_j \) of the equalizer, as shown in Figure 5.3, which minimize the mean square error (MSE) value of the error \( e_k = I_k - \hat{I}_k \).

In most cases, we use optimization techniques, which seek to minimize the MSE, \( E[e_k^2] \) having as a starting point an assumed receive filter structure of the form of (5.61), whose optimal design parameters are determined by the optimization algorithm that is developed.

In order to present how the optimization techniques would work in the design of receiver filters, we can consider a binary PAM system. The transmitted continuous-time signal can be expressed as

\[
s(t) = \sum_{i=-\infty}^{\infty} a_i^0 h_T(t - iT_i) + i_T(t)
\]

(5.63)

where \( a_i^0 \in [-A, A] \) are the transmitted PAM symbols of the desired channel, which are assumed to be statistically independent.

**Figure 5.3** Linear filter equalizer. (After: [7].)
\( i_T(t) \) is the interference from \( 2N \) adjacent channels, with a spacing of \( B_C \) Hz expressed as follows

\[
i_T(t) = \sum_{\ell=-N}^{N} e^{j(2\pi B_C t + \varphi_\ell)} \sum_{i=-\infty}^{\infty} a_{i\ell} h_T(t - iT - \tau_\ell)
\]  

(5.64)

where \( a_{i\ell}, \varphi_\ell, \tau_\ell \) are the \( i \)th symbol, the phase shift, and the delay of the \( \ell \)th adjacent channel, respectively.

The transmitted signal, \( s(t) \), goes through a linear channel that has impulse response, \( c(t) \), and it is also corrupted by Gaussian noise, \( n(t) \). We assume that the receiver filter has impulse response \( h_R(t) \), shown in Figure 5.4.

The output of the receiver filter is then given by

\[
s_o(t) = h_R(t) * \left[ c(t) * \left[ \sum_{i=-\infty}^{\infty} a_i^0 h_T(t - iT) + i_T(t) \right] + n(t) \right]
\]  

(5.65)

\[
= \sum_{i=-\infty}^{\infty} a_i^0 g(t - iT) + i_R(t) + n_R(t)
\]

where

\[
g(t) = h_T(t) * c(t) * h_R(t)
\]  

(5.66)

**Figure 5.4** Linear filter optimization. (After: [7].)
* denotes the convolution operation

\[ n_R = h_R(t) * n(t) \]  \hspace{1cm} (5.67)

\[ i_R(t) = i_T(t) * c(t) * h_R(t) \]

If we assume that the length of the pulse \( g(t) \) is at most \( N = 2M + 1 \) symbols, the signal sampled at \( t = 0 \) can be expressed as

\[ s_0(0) = \sum_{i=-M}^{M} a_i^0 g(-iT) + i_R(0) + n_R(0) = a^T g + i_R(0) + n_R(0) \]  \hspace{1cm} (5.68)

where the vectors \( a \) and \( g \) are given in (5.69).

\[ a^T = [a_{-M} \ldots a_{M-1} \cdot a_M]^T \]  \hspace{1cm} (5.69)

\[ g^T = [g(MT) \ldots g(-(M-1)T) \cdot g(-MT)]^T \]

The error between transmitted symbol \( a_0^0 \) and sampled symbol \( s_0(0) \) using (5.68) is given by:

\[ e_o = a_0^0 - s(0) = a_0^0 - a^T g - i_R(0) - n_R(0) \]  \hspace{1cm} (5.70)

Assuming uncorrelated signal and noise samples, as well as uncorrelated adjacent channel interfering signals, the mean square value of \( e_o \), MSE, is given by squaring (5.70) and finding its mean. This process results in [7]:

\[ E[e_o^2] = A^2 (1 - g(o))^2 + A^2 \sum_{i=-M, i \neq 0}^{M} (g(iT))^2 + \sigma_{ACI}^2 + \sigma_N^2 \]  \hspace{1cm} (5.71)

where

\[ A^2 = E\{a_0^2\}; \]

\[ \sigma_{ACI}^2 = \text{average power of adjacent channel interference (ACI)}; \]

\[ \sigma_N^2 = \text{noise variance}. \]
Our objective is to design a discrete-time receive filter to minimize MSE. We shall further assume that $L$ samples are taken per symbol interval ($1/f_s = T/L$).

The receive filter coefficients will be defined as

$$h_R = [b_R(-M_R) \ldots b_R(M_R-1) b_R(M_R)]^T$$ (5.72)

where the receiver filter coefficients can be expressed as a length $N_R = 2M_R + 1$, whereas similarly the coefficients of the combined transmit filter and channel response will be given by

$$b_{TC}(\kappa) = b_T(\kappa) * c(\kappa)$$ (5.73)

We shall assume that $b_{TC}$ has the following form.

$$b_{TC} = [b_{TC}(-M_{TC}) \ldots b_{TC}(+M_{TC} - 1) b_{TC}(M_{TC})]^T$$

where again $N_{TC} = 2M_{TC} + 1$.

The $k$th sample of $g(t)$ then will be given by

$$g(\kappa) = h_R^T f(\kappa) h_{TC}$$ (5.74)

$f(\kappa)$ is an $N_R X N_{TC}$ swapping matrix performing the discrete convolution and $M_R + M_{TC} \leq ML$.

The second term in (5.71) gives [7]:

$$\sigma_{ISI}^2 = A^2 \sum_{i=-M}^{M} g(iL)^2 = A^2 \sum_{i=-M}^{M} (h_R^T w_i)^2 = A^2 h_R^T W h_R$$ (5.75)

where

$$W = \sum_{i=-M}^{M} w_i \cdot w_i^T = W = f(iL) h_{TC}$$

Similarly

$$\sigma_n^2 = h_R^T R_n h_R$$
where \( R_n = \text{covariance matrix of the noise whose elements } R_{ni}^m \) are given by:

\[
R_n^{\frac{1}{2}} = \int_{-1/2}^{1/2} S_n(\rho) \cos(\ell - m) 2\pi \rho \, d\rho, \text{ where } \rho = f/f_s \quad (5.76)
\]

\( S_n(\rho) = \text{power spectrum of noise} \)

The variance of ACI is given by

\[
\sigma_{ACI}^2 = \int_{-1/2}^{1/2} S_I(\rho) |C(\rho)H_R(\rho)|^2 \, d\rho \quad (5.77)
\]

where

\( S_I(\rho) \) is the power spectrum of ACI;

\( C(\rho) \) is the Fourier transform of the channel response, \( c(t) \);

\( H_R(\rho) = \sum_{i=-M_R}^{M_R} b_R(i) e^{-j2\pi i} \) (Fourier transform of receive filter).

If we define

\[
S_I(\rho) = \sum_{k=-\infty}^{\infty} r_I(\rho) e^{-j2\pi k\rho}, \text{ where} \quad (5.78)
\]

\[
r_I(\kappa) = E[i_T(nT) i_T^*(n + \kappa) T] \quad (5.79)
\]

Using (5.64) and (5.78) and assuming \( E\{\varphi_\ell\} = E[\tau_\ell] = 0, E = \{\alpha_\ell^\ell, \alpha_j^\nu\} = A^2 \) if \( \ell = \nu \) and \( i = j \), otherwise zero, we obtain the power spectrum of ACI as

\[
S_I(f) = A^2 \sum_{\ell=-p}^{p} |H_T(f + \ell B_c T)|^2 \quad (5.80)
\]

From (5.77) we obtain
\[ \sigma_{ACI}^2 = A^2 h_R^T R_{ACI} h_R \] (5.81)

where \( R_{ACI} \) is the covariance matrix of ACI with elements

\[
R_{ACI}(k, \ell) = \int_{-1/2}^{1/2} \left[ \sum_{n=-P}^{P} |H_T(\rho + nB_c T)|^2 \right] |C(\rho)|^2 \cos \left[ 2\pi(k-\ell)\rho \right] d\rho
\] (5.82)

Having expressed all of the terms of \( E[e_0^2] \) in terms of \( h_R \), we can form the following cost functional

\[
Q(h_R, \lambda) = \beta_{ISI} \bar{h}_R^T W h_R + \beta_{ACI} A^2 h_R^T R_{ACI} h_R + h_R^T R_n h_R + \lambda \left[ \bar{h}_R^T w_o - 1 \right]
\] (5.83)

where \( \lambda \) is a Lagrange multiplier, \( \beta_{ISI} \) and \( \beta_{ACI} \), are weight parameters, depending on what emphasis we want to place on ISI and/or ACI.

Taking the derivative of (5.83) with respect to \( h_R \) and setting it to zero, we obtain the optimal value of the design receive filter parameters, \( h_R^* \).

\[
h_R^* = \frac{P^{-1} w_o}{w_o^T P^{-1} w_o}
\] (5.84)

where

\[ P = \beta_{ISI} A^2 W + \beta_{ACI} A^2 R_{ACI} + R_n \] (5.85)

Having determined \( h_R^* \), which is the vector that contains the filter design parameters, we can now proceed to construct the receive filter, which in turn will minimize the error between transmit and receive symbols. In other words, the design parameters of this filter have been obtained, which in turn minimize simultaneously intersymbol and adjacent channel interference. The great advantage of this formulation is that it leads to an optimal design that takes into consideration simultaneously intersymbol, adjacent channel interference, and noise factors. The procedure described earlier, if it is seen
from a different angle, it is equivalent to having led to an optimization of carrier to interference ratio, $C/I$. Over the years, $C/I$ has been used as a measure of performance of wireless systems and as such has also been used lately for cases which deal with resource allocation in wireless communication systems.

### 5.2.5 Near End to Far End Ratio Interference

One type of interference, which occurs only in mobile communication systems, is the near end to far end type of interference [9]. That kind of interference appears when the distance between a mobile unit and the base station transmitter becomes critical with respect to another mobile transmission that is close enough to override the desired base station signal. This phenomenon occurs when a mobile unit is relatively far from its desired base station transmitter at a distance $d_1$, but close enough to its undesired nearby mobile transmitter at a distance $d_2$ and $d_1 > d_2$. The problem in that situation is whether the two transmitters will transmit simultaneously at the same power and frequency, thus masking the signals received by the mobile unit from the desired source by the signals received from the undesired source. Also, this type of interference can take place at the base station when signals are received simultaneously from two mobile units that are at unequal distances from the base station. The power difference due to the path loss between the receiving location and the two transmitters is called the *near end to far end ratio interference* and is expressed by the ratio of path loss at distance $d_1$ to the path loss at distance $d_2$.

This form of interference is unique to the mobile radio systems. It may occur both within one cell or within cells of two systems.

**In One Cell**

When mobile station A is located close to the base station, and at the same time mobile station B is located far away from the same base station (e.g., at the cell boundaries), mobile station A causes adjacent-channel interference to the base station and mobile station B (Figure 5.5). The $C/I$ at mobile station B is expressed by the following equation [9]:

$$\frac{C}{I} = \left(\frac{d_0}{d_1}\right)^\gamma$$  \hspace{1cm} (5.86)

where $\gamma$ is the path loss slope.
Because $d_0 > d_1$, from (5.86) we obtain $C/I < 1$. This means that the interfering signal is stronger than the desired signal.

This problem can be rectified if the filters used for frequency separation have sharp cut-off slopes. The frequency separation can be expressed as follows [9]:

$$\text{frequency band separation} = 2^{G-1} B$$

where

$$G = \frac{\gamma \log_{10} \left( \frac{d_0}{d_1} \right)}{L}$$

$B$ = the channel bandwidth;
$L$ = the filter cut-off slope.

**In Cells of Two Systems**

If two different mobile operators cover an area, adjacent-channel interference may occur if the frequency channels of the two systems are not properly coordinated.

In Figure 5.6, two different mobile radio systems are depicted. Mobile station A is located at the cell boundaries of system A, but very close to base station B. Also, mobile station B is located at the cell boundaries of system B, but very close to base station A. Interference may occur at base station A from mobile station B and at mobile station B from base station A. The same interference will be introduced at base station B and at mobile station A.
This form of interference can be eliminated if the frequency channels of the two systems are properly coordinated, as mentioned earlier. If such a case occurs, two different systems operating in the same area may have colocated base stations.

### 5.3 Interference Analysis Methodology

One of the main design goals in the cellular mobile terrestrial and satellite communication systems is to provide high capacity in combination with the required quality of service. Due to the architectural structure of these systems, a very crucial issue is the determination methodologies for analyzing the nature and the influence of any kind of interference. Up to now, the system designer almost always assumed that the limiting corrupting signal has Gaussian characteristics, such as the characteristics of thermal noise. With the advent of low-noise receivers and congestion in the radio frequency bands, this assumption can no longer be justified, and interference of non-Gaussian nature into our present and future communication systems is an important issue. The method of analysis used to determine the effect of thermal noise on communication systems cannot, therefore, be used blindly to determine the effect of interference of non-Gaussian nature on the new and evolving wireless systems and thus to design system components. Various analysis tools have been developed, which take into consideration interference not only as an additive distorting agent but also as a multiplicative agent, as in fading, as we saw in Chapter 4. The main objective is then to analyze...
how the interference as a general distortion agent affects well-accepted criteria of performance of wireless systems, such as $C/I$ or $S/I$ and BER, and then proceed to develop optimal or suboptimal design tools that lead to practical system implementation and that satisfy predetermined minimum performance levels. Chapter 6 will do just that, and Chapter 7 will show how the results of Chapters 5 and 6 will be used to design practical implementations, which satisfy set goals of performance. The analysis methodology that is involved in order to achieve our objective is presented in the following sections. It takes, as a basic analysis tool, the determination of the $C/I$, $S/I$, or BER as functions of critical design parameters. The reader is encouraged to refer to the Preface of this book to appreciate the importance of the methodology developed here as a part of the overall methodology developed in the beginning of this book to cover and justify the relevance and interrelationship of all chapters.

This methodology consists of the following steps:

1. Calculation or estimation of interference power density;
2. Calculation of $C/I$ power ratio;
3. Determination of relationship between $C/I$ and $S/I$ or error probability ($P_e$);
4. Determination of relationship between $S/I$ or $P_e$ and system performance;
5. Determination of relationship between system performance and acceptable level of system parameter changes for improving system performance;
6. Use of $C/I$ as a measure for the optimization of resource allocation and quality;
7. Develop mechanisms and criteria for interference reduction. In any case, develop methods that calibrate the affect of interference by manipulating design parameters.

The two parameters $C/I$ and $S/I$ as a quality measure are intimately related with the grade of service of the wireless systems and for the case of cellular systems with the following parameters:

- Carrier to cochannel interference ratio;
- Blocking probability.
Over the years, practical values for these parameters have been obtained, which set the quality criteria for specific practical wireless systems in use.

5.3.1 Analog Signals

Analog signals are those signals that are produced by the information source (voice or image) and are used for transmission in analog form (i.e., continuous in time). Even though most of the information signals used nowadays for transmission are either digitized (digital) or are produced by the source in data form, we still need to discuss and analyze their interference aspects because the development of interference reduction techniques of digital signals are mostly based on these classical schemes, as we shall see in Chapter 6.

Essential for computing the baseband interference is knowledge of the RF power spectral densities of both the desired and interfering signals. Let the desired angle-modulated signal $s_1(t)$ and an arbitrary narrowband interfering signal $s_2(t)$ be given by [1–9]:

\[
\begin{align*}
  s_1(t) &= \text{Re} \left[ z_1(t) \right] = \text{Re} \left[ A_1 \exp \left( j \omega_1 t + x_1(t) + \mu \right) \right] \quad (5.87) \\
  &\quad = \text{Re} \left[ A_1 u_1(t) \left( \exp j \omega_1 t \right) \right] \\
  s_2(t) &= \text{Re} \left[ z_2(t) \right] = \text{Re} \left\{ V_2(t) \exp \left( j \omega_2 t \right) \right\} \quad (5.88)
\end{align*}
\]

respectively.

It is assumed that $s_1(t)$ and $s_2(t)$ are both wide-sense stationary and are generated from separate sources; thus, they are statistically independent of each other. Furthermore, $x_1(t)$ and $\mu$ are assumed to be independent and $\mu$ is assumed to be uniformly distributed in $\{0–2\pi\}$. At the input of a demodulator, both signals are added and go through a phase detector, assuming we’re dealing with phase modulated signals. The sum of these two signals is given by

\[
  s(t) = s_1(t) + s_2(t) = \text{Re} \left( a(t) \right) \exp \left( j \omega_1 t + x_1(t) + \lambda(t) \right)
\]

where

\[
a(t) e^{j\lambda(t)} = 1 + z(t) \exp \left( j (\omega_2 - \omega_1) t - x_1(t) + \mu \right)
\]

and

\[
z(t) = \frac{z_2(t)}{z_1(t)}
\]
In [1], it is shown that under certain mild conditions, the output of the demodulator will contain the desired signal plus the excess phase caused by the interfering signal. The excess phase angle (caused by the presence of the interference) at the output of an ideal demodulator is given by

\[ \lambda(t) = \text{Im} \ln \left[ 1 + \frac{z_2(t)}{z_1(t)} \right] \]  \hspace{1cm} (5.89)

For \(|z_2(t)/z_1(t)| < 1\), \(\lambda(t)\) can be expanded as

\[ \lambda(t) = \text{Im} \sum_{m=1}^{\infty} \frac{(-1)^{m+1}}{m} \left( \frac{z_2(t)}{z_1(t)} \right)^m = \sum_{m=1}^{\infty} \lambda_m(t) \]  \hspace{1cm} (5.90)

The baseband power spectrum of the demodulated interference is obtained from the autocorrelation function of the total detected phase \(\phi(t)\) where

\[ \phi(t) = x_1(t) + \lambda(t) \]  \hspace{1cm} (5.91)

the autocorrelation function is thereby given

\[ R_\phi(\tau) = \langle [x_1(t) + \lambda(t)] \cdot [x_1(t + \tau) + \lambda(t + \tau)] \rangle = R_{x_1}(\tau) + R_\lambda(\tau) \]  \hspace{1cm} (5.92)

Because the cross terms vanish when averaged over \(\mu\), the \(m\)th term of \(\lambda(t)\) can be written as

\[ \lambda_m(t) = \text{Im} \left\{ V_2^m(t) \exp(jm\omega_2 t) \exp[jx_m(t)] \right\} K_m \]

\[ \lambda_M(t) = \frac{K_m}{2j} \left\{ V_2^m(t) \exp[jm\omega_2 t] \exp[jx_m(t)] \right\} \]

\[ - V_2^m(t)^* \exp[-jm\omega_2] \exp[-jx_m(t)] \]  \hspace{1cm} (5.93)

where

\[ x_m(t) = -m[\omega_1 t + x_1(t) + \mu] \]  \hspace{1cm} (5.94)

and
The term $A_1$ represents the wanted carrier amplitude.

Equation (5.93) can be used to find the PSD of $\lambda_m(t)$ and the autocorrelation function of $\lambda_m(t)$, $R_{\lambda m}^m(\tau)$. It is shown that [5]:

$$R_{\lambda m}^m(\tau) = \left[ \frac{1}{4m^2 A_1^{2m}} R_{V_2^m}(\tau) R_{u_1^m}^*(\tau) \exp [jm(\omega_2 - \omega_1)\tau] \right. $$

$$+ \left. R_{V_2^m}^*(\tau) R_{u_1^m}^m(\tau) \exp [-jm(\omega_2 - \omega_1)\tau] \right]$$ (5.96)

where the $R_{V_2^m}^m(\tau)$ is the autocorrelation function of $V_2^m(t)$, and the $R_{u_1^m}^m(\tau)$ is the complex conjugation of the autocorrelation function of $u_1^m(t)$.

The power spectrum of the baseband interference is then given by

$$I(f) = \sum_{m=1}^{\infty} \frac{1}{4m^2 A_1^{2m}} [T_m(f - mf_i) + T_m(-f - mf_i)]$$ (5.97)

where

$$T_m(f) = S_{V_2^m}(f) \otimes S_{u_1^m}(f)$$ (5.98)

with

$$S_{V_2^m}(f) = F[R_{V_2^m}(\tau)] = \text{power spectral density of } V_2^m(t);$$

$$S_{u_1^m}(f) = F[R_{u_1^m}(\tau)] = \text{power spectral density of } u_1^m(t).$$

where

* denotes complex conjugate;

$\otimes$ denotes convolution.

The solution to the problem of interference into an angle-modulated system in its most general form therefore comprises two convolution terms. Convolving the power spectral of the $m$th power of the complex envelopes can generate each term. These spectral densities will be used to calculate C/I.
5.3.1.1 Calculation of $C/I$

For simplicity, we shall assume that the transmitted modulated analog signal is given by the following equation:

$$s(t) = A \cos (\omega_1 t + \varphi(t))$$  \hspace{1cm} (5.99)

and the interference is expressed by

$$i(t) = R(t) \cos (\omega_2 t + \psi(t) + \mu) = \text{Re} \{u(t) \exp (j \omega_2 t + \mu)\}$$  \hspace{1cm} (5.100)

where

$\varphi(t)$ includes the information signal;

$\psi(t)$ includes the interference signal;

$\mu$ is assumed to be uniformly distributed on $[0, 2\pi]$.

It is assumed that at the receiver, we obtain the sum of these two signals, which is indicated as $s_0(t)$ and is given by the following formula

$$s_0(t) = s(t) + i(t)$$  \hspace{1cm} (5.101)

Equation (5.101), using (5.99) and (5.100), can be written as shown in (5.102) using simple trigonometric identities

$$s_0(t) = \text{Re} \left( Aa(t) e^{j(\omega_1 t + \varphi(t) + \lambda(t))} \right)$$  \hspace{1cm} (5.102)

where

$$\lambda(t) = \text{Im} \ln \left( 1 + z(t) e^{j(2\pi f_\Delta t - \varphi(t) + \mu)} \right)$$  \hspace{1cm} (5.103)

and

$$a(t) e^{j\lambda(t)} = 1 + z(t) e^{j(2\pi f_\Delta t - \varphi(t) + \mu)}$$  \hspace{1cm} (5.104)

$$z(t) = \frac{u(t)}{A}, f_\Delta = f_2 - f_1$$  \hspace{1cm} (5.105)
Interference Analysis

\[ f = \frac{\omega}{2\pi} \]  

(5.106)

If we assume \(|z(t)| \ll 1\), thus (5.91) can be expanded in a series given by

\[ \lambda(t) = \sum_{m=1}^{\infty} \frac{(-1)^{m+1}}{m A^m} (R(t))^m \sin(m2\pi f \Delta t - \varphi(t) + \psi(t) + \mu) \]

If the receiver we use for detection is an ideal phase detector, as shown in Figure 5.7, we obtain as output \(\phi(t) + \lambda(t)\).

We observe that the contribution of the interference signal \(i(t)\) to the transmitted information signal \(s(t)\) is the signal \(\lambda(t)\).

In order to determine the level of performance deterioration for analog signal transmission, a criterion of performance measured in decibels has been developed that is given by

\[ 20 \log \left( \frac{S}{I} \right) \]  

(5.107)

This is 20 times the logarithm of the ratio of the signal power to interference power. We need, therefore, to calculate the power ratio \(S/I\).

For any given \(\phi(t)\) and \(\psi(t)\), the calculation of this power ratio is very difficult, and we usually use some approximation. In most cases of analog transmission, this yields acceptable results.

For example, for typical multichannel frequency division multiplex frequency modulated telephony signals, both \(\phi(t)\) and \(\psi(t)\) can be assumed to be Gaussian independent and stationary. With these assumptions, it can be shown [5] that

\[ \frac{S}{I} = \frac{(2\pi)^2 M^2 \sigma^2}{r^2(1 - \varepsilon)} \left[ \int_{f_0 - \frac{b}{2}}^{f_0 + \frac{b}{2}} (2\pi f)^2 \frac{S_\lambda(f)}{f} \, df \right]^{-1} \]  

(5.108)

Figure 5.7 Model of an ideal phase detector.
where

\[ f_c = \text{center frequency of channel under construction}; \]
\[ b = \text{telephone channel bandwidth}; \]
\[ f_{m_1} = \text{top baseband frequency of want signal}; \]
\[ M_I = r_{MS} \text{ modulation index of wanted multichannel baseband}; \]
\[ \epsilon_1 = \text{ratio of lowest to highest frequency of multichannel baseband}. \]

We observe that if this ratio is not acceptable, it can be changed by changing the appropriate parameter in (5.108). The reader can realize that for cases when the assumptions taken for the calculation of (5.108) don’t hold, the derivation of an equation equivalent to (5.108) may not be possible in closed form. In such cases, we have to resort to various computational methods and simulation techniques. In any case, for any type of analog modulation used and for any type of service, such as voice or TV implemented, the relationship between performance deterioration with \( S/I \) is given to the system designer beforehand. The current approach for very complex analog systems is to use heuristic methods, such as neural networks for the determination of \( S/I \) and other design parameters of wireless systems [10].

If we go back to (5.108), we observe that \( S/I \) depends directly on \( 1/r^2 \), which is the carrier power \( A^2/2 \) to interference \( r^2 A^2/2 \) ratio denoted by \( C/I \). Hence,

\[
\frac{C}{I} = \left( \frac{1}{r^2} \right) = \frac{\frac{A^2}{2}}{r^2 \frac{A^2}{2}} \quad (5.109)
\]

In other words, in all cases for any modulation system under consideration, we have a priori:

\[
\frac{S}{I} = R \frac{C}{I} \quad (5.110)
\]

where \( R \) is a constant. For our case, under the assumptions made, this constant is given by (5.108) if we factor out \( 1/r^2 \).
For the case of satellite systems, it can be shown [1] that the C/I is a function of intersatellite spacing, $\Delta \theta$. For certain modulation systems used and for certain services provided by a satellite system, setting a specific level of quality for the service provided, we determine the value of the required $S/I$ and thus $C/I$. This specific value of $C/I$ sets a limit on the intersatellite spacing and thus on the orbit utilization. It is therefore important to realize that for satellite systems, the interference plays a major role in the orbit utilization. Coupled with thermal noise and for a specified limit of total noise into a certain channel, interference is one of the major factors of orbit utilization in satellite systems.

### 5.3.2 Digital Signals

For the case of digital systems, we shall take a standard PSK signal of the form [3–8]:

$$s(t) = A \cos (\omega_1 t + \varphi_1(t))$$  \hspace{1cm} (5.111)

For simplicity, we choose $A = 1$. The digital modulation is carried in the angle of $s(t)$ by $\phi_1(t)$, which assumes discrete values from a set of $M$ equally spaced points in $[0, 2\pi]$ at the sample times $T$ seconds apart. Thus the $N$th message or baud is modulated by

$$\varphi_1(NT) = \frac{2\pi k}{M}, \; k = 0, 1, 2, \ldots, M - 1$$

where each of $M$ values of $k$ is equally probable. For a coherent receiver, which compares the received wave with the unmodulated carrier, $A \cos \omega_1 t$, and produces instantly the signed phase difference between the two points, an M-ary symbol is transmitted in one baud by the value of $k$.

The external mainly thermal noise is modeled in the usual fashion by a stationary zero mean Gaussian random process with uniform spectral density, as mentioned in the previous section. Hence,

$$n(t) = n_1(t) \cos \omega_1 t - n_2(t) \sin \omega_2 t$$  \hspace{1cm} (5.112)

where $n_1(t)$ and $n_2(t)$ are stationary independent, zero mean Gaussian random processes with power $\sigma^2$. 


The interference signal shall be modeled by

\[ i(t) = rA \cos(\omega_2 t + \varphi_2(t) + \mu) \quad (5.113) \]

At a certain instant, the combined input signal at the detector is given by

\[ s'(t) = s(t) + n(t) + i(t) \]

and is presented in Figure 5.8.

The detector examines the difference between the phase of the received signal and the reference phase and decides which symbol was transmitted. Assuming equal a priori symbol probabilities, for a proper decision we need to define decision thresholds by dividing the circle into regions, as shown in Figure 5.9 for the case of \( M = 8 \).

\[ \frac{\pi}{M}, \frac{3\pi}{M}, \ldots, \frac{(2M - 1)\pi}{M} \]

Therefore, at the instant of detection, if the phase of the received signal lies within the region, \( 0 \leq \theta \leq \pi/4 \), we make the decision that the symbol, having been transmitted, corresponds to the value \( k = 1 \).

5.3.2.1 \( C/I \) or \( S/I \) as a Performance Measure—Digital Signals

In the previous section, we defined the \( C/I \)s and SNRs and we said that, depending on the service to be offered by the system designed, certain values

![Figure 5.8 Phasor diagram of the signal, noise, and interference.](image-url)
for these ratios are specified beforehand. These values are usually determined by qualitative evaluation of the service offered. For digital signals, the qualitative evaluation depends on the number of errors the system causes to the received data operating in a particular interference environment. Having thus set the values of these ratios for satisfactory quality of service, we can use them as references and thus can consider these ratios as quality measures.

1) PSK Systems

In order to develop a measure of performance for digital systems, as we saw in the previous section for analog systems, it is customary to seek to establish a relationship between the ratio of the transmitted signal carrier power to interference power and the probability of error. By probability of error, we understand the probability that the angle \( \phi \) is outside of the decision region.

The coordinates of \( \theta (x; y) \), which are random variables, have means given next.

\[
\begin{align*}
\bar{x} &= r \sin \varphi \\
\bar{y} &= 1 + r \cos \varphi
\end{align*}
\]  

(5.114)

conditioned, of course, on the angle \( \phi \). We also see \( A = 1 \) for simplicity. The conditional joint PDF of \( x, y \) is given by

\[
f_{XY}(x, y | \varphi) = \frac{1}{2\pi\sigma^2} e^{-\frac{1}{2\sigma^2}[(x-r \sin \varphi)^2+(y-1-r \cos \varphi)^2]}
\]

(5.115)
If we multiply (5.115) by the PDF of $\phi$, which is $1/2\pi$ integrate over the interval $[0, 2\pi]$ we obtain [1–3]:

$$f_{XY}(x, y) = \frac{-\frac{1}{2\sigma^2}(x^2+(y-1)^2+r^2)}{(2\pi\sigma)^2} \int_0^{2\pi} e^{\frac{-r}{\sigma^2}(x^2+(y-1)^2)^{1/2} \cos(\varphi+\eta)} \ d\varphi$$

(5.116)

where

$$\eta = \tan^{-1} \frac{y-1}{x}$$

Equation (5.116) gives

$$f_{XY}(x, y) = e^{\frac{-\frac{1}{2\sigma^2}(x^2+(y-1)^2+r^2)}{(2\pi\sigma)^2}} I_o \left( \frac{r}{\sigma^2} (x^2 + (y-1)^2)^{1/2} \right)$$

(5.117)

where

$I_o$ is the zero modified Bessel function of the first kind.

Equation (5.117) gives the joint PDF of the components of the received signal. We need, however, the PDF of the received signal phase. We achieve our objective if we change the variable $x, y$ into polar coordinates and integrate over the phasor’s length. If we set

$$x = \delta \sin a$$

$$y = \delta \cos a$$

(5.118)

Equation (5.117) becomes

$$f_{\theta}(\vartheta) = \frac{1}{(2\pi\sigma)^2} \int_0^\infty e^{\frac{-\frac{1}{2\sigma^2}((\delta^2+r^2)+1-2\delta \cos a)}{(2\pi\sigma)^2}} I_o \left( \frac{r}{\sigma^2} (\delta^2 + 1 - 2\delta \cos a)^{1/2} \right) \ d\delta$$

(5.119)
If we now integrate equation (5.119) over the region, which lies outside the boundaries from $-\pi/M$ to $\pi/M$, we obtain

$$\text{Probability of error} = P_e = 2 \int_{(\pi/M)}^{\pi} f_{\Theta}(\theta) \, d\theta \quad (5.120)$$

We use the factor 2 because $f_{\Theta}(\theta)$ is symmetric with respect to $\theta$. A graphical representation of (5.120) is given in the Figure 5.10 for $M = 4$ [3].

We observe from (5.120) and Figure 5.10 that $P_e$ depends directly on the parameters $1/r$ and $1/\sigma$, which are the $C/I$s and CNRs. Having related probability of error, which is a quality measure, to the $C/I$ with design parameters, our analysis has led to our original objective to relate quality measures to design objectives in any type of interference environment. The remaining sections will be devoted to calculating $C/I$ for other types of applications of wireless systems. In the following sections we shall apply this analysis to other cases of interference.

2) Terrestrial Mobile Cellular Communications Systems

In this section we shall present a methodology used to calculate the $C/I$ for cellular and mobile systems. This methodology will then be applied to calculate $C/I$ for standard mobile systems currently in use.

The $C/I$ of a cellular system can be approximated by [9, 11].

$$\frac{C}{I} = \frac{1}{M} \cdot \left(\frac{D}{R}\right)^n \quad (5.121)$$

where

- $M$ = the number of cochannel interfering cells;
- $n$ = a path loss exponent that ranges between two and four in urban cellular systems;
- $D$ = distance between two cochannel cells;
- $R$ = the radius of a cell.

a) TDMA Cellular

For TDMA cellular networks, the mean $C/I$ at any given location is given by [12–14]:

$$\frac{C}{I} = \frac{1}{M} \cdot \left(\frac{D}{R}\right)^n$$
$C/I = 10 \log \left[ \frac{S_d}{\sum_{i=1}^{n} I_i} \right]$ \hspace{1cm} (5.122)

where

$S_d = \text{the desired signal strength};$

$I_i = \text{the interference from the } i\text{th cochannel base station}.$
Calculation of Signal to Interference Plus Noise Ratio for TDMA

Many contemporary cellular radio resource management algorithms for hand-offs, channel assignment, and power control assume fast and accurate measurements for the signal to interference plus noise ratio $S/(I + N)$.

Several methods have been recently developed to generate real-time estimates of the $S/(I + N)$ in TDMA cellular systems:

1. Interference projection (IP), which uses the training and/or color code sequences that are typically present within cellular TDMA slots to obtain an unbiased estimate of $S/(I + N)$.

2. Use of the autocorrelation sequence of the received signal samples over a short time scale.

3. Subspace-based (SB) estimates of $S/(I + N)$ obtained by the use of the eigenvalues of the co-variance matrix of the received signal sequence.

4. Use of signal to variation power (SVP) estimator. This method uses the autocorrelation sequence of the received signal samples for a short time scale. However, numerical results (in DECT SYS) reveal that the estimator suffers from a large bias for interesting values of the $S/I$.

5. Signal projection (SP) methods have a computational complexity comparable to the IP methods and an average absolute $S/(I + N)$ prediction error comparable to the SB methods.

b) OFDM/CDMA

The basic equation for the $C/I$ of a user and a carrier for an OFDM/CDMA system in the case of synchronously arriving signals is [11]:

$$ (C/I)_i = \frac{P_{iR} G_p}{\sum_{j=0}^{N} a_j \cdot P_{jR} + \sum_{k=0}^{M} \beta_{IC} \cdot P_{kR}^{tot} + N_0} \quad (5.123) $$

where
\(P_{iR}\) = the receiver power of the carrier \(i\);

\(P_{tot}^{kR}\) = the total received power from base transceiver station (BTS) \(k\);

\(G_p\) = the processing gain;

\(a_j\) = the orthogonality factor for intracell interference;

\(\beta_{IC}\) = models the orthogonality loss due to nonideal channel estimation and due fading multipath channel;

\(N_0\) = models the thermal noise.

The equation for single carrier is given as

\[
(C/I)_{i} = \frac{P_{iR} \cdot G_p}{\left[ \sum_{k=0}^{M} \beta_{IC} \cdot P_{tot}^{kR} \right] \cdot \gamma} \tag{5.124}
\]

The equation for a single user will now be

\[
(C/I)_{i} = \frac{P_{R} \cdot G_p}{\left[ \sum_{k=0}^{M} \beta_{IC} \cdot P_{tot}^{kR} \right] \cdot \gamma} \text{ where } P_{R} = \sum_{i=0}^{N} \frac{P_{iR}}{N} \tag{5.125}
\]

The parameter \(\gamma\) models the orthogonality between the signals from different BTS.

c) CDMA Cellular Systems

To maintain the communications quality in CDMA cellular systems at the target level, \(S/I\)-based power control methods have been proposed [12–15]. In the uplink, all MSs in a cell control their transmission power so that the received power attains the desired power level at the connecting base station. In the downlink, a base station allocates its transmission power so that the MSs in the cell have the same \(S/I\). Therefore, all MSs in a cell have the same uplink \(S/I\) and the same downlink \(S/I\), as shown in Figure 5.11.

Uplink communication quality at \(BS_0\), \(SIR_{0\_up}\), is expressed as

\[
SIR_{0\_up} = \frac{P_{R0}}{(N_0 - 1) \cdot P_{R0} + B_0} = \frac{1}{(N_0 - 1) \cdot B_0 / P_{R0}} \tag{5.126}
\]
Here $P_{R0}$ represents the desired power level at $BS_0$ and becomes the target for transmission power control when MSs connect to $BS_0$. The first term in the denominator is the interference from other MSs in the same cell. The second term expresses the interference from other cells, and it’s denoted as $B_0$.

Downlink communications quality of $MS(0, j)$ connected to $BS_0$, $SIR(0, j)_{down}$ is expressed as

$$SIR(0, j)_{down} = \frac{P_A(0, j) \cdot L_0(0, j)}{(1 - F_0) \cdot P_{BS}} \cdot L_0(0, j) + C(0, j) \quad (5.127)$$

The signal sent from $BS_0$ to $MS(0, j)$ is transmitted with a power of $P_A(0, j)$. The propagation loss between $MS(0, j)$ and $BS_0$ is represented as $L_0(0, j)$ [15].

The denominator at the right-hand side in (5.127) represents the total interference at $MS(0, j)$. The total transmission power at $BS_0$ is expressed as $P_{BS}$. $C(0, j)$ is the total interference from the other cells at $MS(0, j)$. We define an orthogonality factor $F_0$ in the downlink, and thus $(1 - F_0)$ represent the degree of loss in orthogonality. The orthogonality factor depends on such characteristics as the number of propagation paths, the power ratio between paths, and the number of fingers in the RAKE receiver (see Figure 5.11).

The downlink SIR at $BS_0$, $SIR_{0\_down}$ is expressed as

$$SIR_{0\_down} = \frac{P_{BS} - P_{pl}}{N_0 \cdot (1 - F_0) \cdot P_{BS} + \sum_{j=1}^{N_0} \frac{C(0, j)}{L_0(0, j)}} \quad (5.128)$$

where $P_{pl}$ indicates the pilot-signal transmission power.

**Figure 5.11** Example of possible links in a cellular system. (After: [9].)
From (5.128), the communication quality in the downlink is affected by the following factors:

- Number of MSs in the target cell $N$;
- The total transmission power $P_{BS}$;
- The orthogonal factor $F_0$;
- The interference from other cells $C(i, j)$;
- The propagation loss $L(i, j)$.

d) Macrocell and Microcell Systems
Without power control for downlink, the transmitted power of the BS to MS located anywhere, is the same. The $C/I$ experienced by a mobile in the central macrocell and in the microcell can be derived as [15, 16]

$$ \left[ \frac{C}{I} \right]_l = \frac{p_{dl}(1 - a_l) \cdot L_p}{1 - \frac{(1 - a_l)}{N}} \cdot:\ \left[ p_{dl} \cdot L_p + p_{ls} \cdot L'_p + \sum_{i=1}^{6} p_{dl} \cdot L'_{pi} \right] \quad (5.129) $$

$$ \left[ \frac{C}{I} \right]_S = \frac{p_{ls}'(1 - a_s) \cdot L'_p}{1 - \frac{(1 - a_s)}{M}} \cdot:\ \left[ p_{ls}' \cdot L'_p + p_{dl} \cdot L_p + \sum_{i=1}^{6} p_{dl} \cdot L'_{pi} \right] \quad (5.130) $$

where

$p_{dl}$ and $p_{ls}$ = the transmitted power from the macrocell BS and microcell BS;

$L_p$, $L'_p$ and $L_{pi}$ are the path loss for macrocell, microcell, and adjacent macrocell, respectively, and $a_l$ and $a_s$ parameters set to certain values (around 0.1) in order to maximize the capacity of macrocell and microcell and

$$ \sum_{i=1}^{6} p_{dl} \cdot L'_{pi} = \text{the interference from the six adjacent macrocell} \quad (5.131) $$
These formulas are the decision rules for downlink to accept a newly active MS.

The total transmission power of macrocell or microcell for users should be less than \((1 - a)\) of total power. That is,

\[
\sum_{i=1}^{N} P(y) \leq p_t(1 - a)
\]

(5.132)

where

\(N\) = the number of users in a dedicated cell;
\(a\) = the pilot power fraction.

With downlink power control applied to a macrocell or a microcell, the \((C/I)\) for mobile \(i\) is modified as

\[
\left[ \frac{C}{I} \right]_d = \frac{f(y_i) \cdot P_R \cdot L_P}{(p_t - f(y_i)P_R) \cdot L_P + p_i' \cdot L'_P + \sum_{i=1}^{6} p_i' \cdot L'_P} \geq \left( \frac{C}{I} \right)_{td} = -16 \text{ db}
\]

(5.133)

where

\(p_t\) = total transmitted power from the BS belonging to the other kind of cell with corresponding (path loss);

\(p_i'\) = transmitted power from each adjacent macrocell BS with corresponding (path loss).

Equation (5.133) represents the decision rule for accepting a newly active MS in the downlink.

e) Carrier to Cochannel Interference Ratio in Mobile \((C/I)\)

In this section, we shall use simplified models of standard cellular mobile systems currently implemented in order to determine \(C/I\) due to cochannel interference. The mobile unit at location \(M\) in Figure 5.12 receives the desired signal on frequency \(F_1\) from the nearest base station. Simultaneously, the mobile unit at \(M\) also receives independent undesirable interfering signals from other base stations on the same frequency. The same receiver receives
these independent signals, all of which are on the same frequency, simultaneously. This results in the presence of cochannel interference.

The frequency reuse distance $D$ is a function of the number $K_0$ of the interfering cells, as well as the $C/I$ ratio at the mobile receiver. This ratio is defined using the following equation [9]:

$$\frac{C}{I} = \frac{C}{\sum_{k=1}^{K_0} I_k}$$

(5.134)

where $I_k$ is the power of the interfering signal originating from the $K$th cochannel cell. The interfering signals originating from base stations other than those belonging in the first tier are considered to be negligible.

It is known that

$$C \propto R^{-\gamma}$$

(5.135)

and

$$I \propto D^{-\gamma}$$

(5.136)
It can be proved that

\[
\frac{C}{I} = \frac{R^{-\gamma}}{K_0} \sum_{K=1}^{K} D_K^{-\gamma}
\]  

(5.137)

where \( R \) is the cell radius.

In the following paragraphs certain cases, where cochannel interference occurs, are presented. Cases using omnidirectional base station antennas, as well as cases using directional antennas of different directivities are described.

\textit{f) System Using Omnidirectional Antennas}

First we will examine the case with a seven-cell cluster, Figure 5.13.

In a cluster with seven cells, in the presence of six interfering cells in the first tier, the \( C/I \) becomes

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{seven-cell_cluster}
\caption{Seven-cell cluster with omnidirectional antennas. (After: [9].)}
\end{figure}
\[
\frac{C}{I} = \frac{R^{-\gamma}}{\sum_{k=1}^{6} D_{k}^{-\gamma}} = \frac{1}{6} \left( \frac{D}{R} \right)^{-\gamma} = \frac{1}{6} \sum_{k=1}^{6} q_{k}^{-\gamma} \tag{5.138}
\]

where \(q_{k}\) is the cochannel interfering reduction factor at the \(k\)th cell.

Assuming that the propagation path loss slope, \(\gamma\), is equal to 4, and all distances \(D_{k}\) are equal to \(D\), (5.138) yields

\[
\frac{C}{I} = \frac{1}{6 \left( \frac{D}{R} \right)^{4}} \tag{5.139}
\]

In a cluster with seven cells, in the presence of six interferers and when the mobile unit is located at the cell boundaries (worst case), the \(C/I\) becomes

\[
\frac{C}{I} = \frac{R^{-\gamma}}{2(D - R)^{-\gamma} + 2(D)^{-\gamma} + 2(D + R)^{-\gamma}} = \frac{1}{2(q - 1)^{-\gamma} + 2(q)^{-\gamma} + 2(q + 1)^{-\gamma}} \tag{5.140}
\]

For \(\gamma = 4\), (5.140) becomes

\[
\frac{C}{I} = \frac{R^{-4}}{6(D - R)^{-4}} = \frac{1}{6(q - 1)^{-4}} \tag{5.141}
\]

This is the case shown in Figure 5.14.

\textit{g) System Using Directional Antennas}

Using directional antennas on the base stations in the architecture of seven-cell cluster, we have the following possible cases.

\textit{Three-Sector Case}

In the case depicted in Figure 5.15, directional antennas of 120° directivity are used.
In the worst case described earlier, \( C/I \) becomes:

\[
\frac{C}{I} = \frac{R^{-4}}{(D + 0.7R)^{-4} + D^{-4}} = \frac{1}{\left(\frac{D}{R} + 0.7\right)^{-4} + \left(\frac{D}{R}\right)^{-4}} = \frac{1}{(q + 0.7)^{-4} + q^{-4}}
\]

(5.142)

**Six-Sector Case**

In this case directional antennas of 60° directivity are used. In the worst case, as shown in Figure 5.16, \( C/I \) becomes

\[
\frac{C}{I} = \frac{R^{-4}}{(D + 0.7R)^{-4} + D^{-4}} = \frac{1}{\left(\frac{D}{R} + 0.7\right)^{-4}} = \frac{1}{(q + 0.7)^{-4}}
\]

(5.143)
Figure 5.15  Worst case in a seven-cell cluster with antennas of 120° directivity.

Figure 5.16  Worst case in a seven-cell cluster with antennas of 60° directivity (After: [9].)
3) Mobile Satellite Systems
The assumption for the C/I calculations in nongeostationary satellite systems are [17, 18]:

- The selected satellite for the communication link is the one from which maximum power is received.
- Any mobile will have the same carrier power at the receiver input. Hence, power control is required to compensate the variation in losses, which are dependent on the mobile location relative to the satellite.
- The mobile antenna is omnidirectional.
- The interferer causes maximum interference when the frequency spectrum is totally overlapped.

Using these assumptions, the C/I equations are as follows:

The $C$ is the carrier power received at the mobile terminal.

$$C = \frac{P_{Tw} G_{TW}(\vartheta) G_{RW}(\alpha)}{L(d) P_{e,am}}$$  \hspace{1cm} (5.144)

The PN density $I_{01}$, is the multiple access interference resulting from $(m - 1)$ interferers (i.e., $m$ users are communicating simultaneously per carrier in each spotbeam) in the same spotbeam and can be written as:

$$I_{01} = \frac{Ca(m - 1) F_1(10)^{\frac{\Delta}{10}}}{B}$$  \hspace{1cm} (5.145)

The PN density $I_{02}$, is the beam-to-beam interference resulting from $m$ interferers in all adjacent spotbeams, assuming frequency reuse of the neighbor cell, and can be written as:

$$C_1 = \frac{G_{T1}(\theta) G_{RW}(\alpha)}{L(d) P_{e,am}}$$  \hspace{1cm} (5.146)

$$I_{02} = \frac{C_1amF_2 \cdot (10)^{\frac{\Delta}{10}}}{B}$$  \hspace{1cm} (5.147)
The $C/I$ levels are added as thermal noise. The total interfering signal power is the sum of the powers from all ‘$N$’ visible satellite spotbeams in the interfering system.

$$\begin{bmatrix} \frac{C}{I_0} \end{bmatrix}_T = \begin{bmatrix} \frac{C}{I_{01}} + \sum_{k=1}^{N} I_{02,k} + I_{03} \end{bmatrix}$$

(5.148)

where

$P_{TW} = \text{wanted satellite spotbeam power};$

$P_{TI} = \text{interfering satellite spotbeam power};$

$G_{TW} (\theta) = \text{wanted spotbeam gain in direction } \theta;$

$G_{TI} (\theta) = \text{interfering spotbeam gain in direction } \theta;$

$G_{RW} (\alpha) = \text{mobile terminal antenna gain in direction } \alpha;$

$L (d) = \text{free space path loss};$

$P_e = \text{propagation effects which takes into account the shadowing (fading loss) for the link—a function of elevation angle and environment};$

$a = \text{voice activity ratio};$

$m = \text{number of users per carrier};$

$\Delta = \text{power control error};$

$F_i = \text{correlation factor};$

$B = \text{subband bandwidth};$

$I_{03} = \text{external interference inband-shared scenario}.$

4) WLL Communications Systems

For the purpose of the analysis here, WLL indicates a system that connects subscribers to the public switched telephone network using radio signals as a substitute for copper for the entire connection between the subscriber and the switch. The physical layout of such a system is shown in Figure 5.17.

We shall assume that the WLL system operates in an area where there exists a regular microwave link of fixed service-frequency division multiplex/
Figure 5.17  WLL application. (After: [19]. © 2001 John Wiley & Sons, Inc.)
frequency modulation (FS-FDM/FM) telephone service and total signal input at the fixed service microwave link (FS-ML) receiver is given by

\[ s(t) = s_d(t) + s_I(t) + n(t) \]

where

\[ s_d(t) = \sqrt{2P_0} \cos(\omega_0 + t\varphi(t)) \]

where

\[ f_c = \frac{\omega_0}{2\pi} \]

\[ P_0 = \text{carrier power}; \]
\[ \omega_0 = \text{carrier frequency}; \]
\[ \varphi(t) = 2\pi \int x_{FDM}(\tau) \otimes h_p(\tau) \, d\tau. \]

where

\( x_{FDM} \) is the modulating FDM signal;
\( h_p(t) \) is the pre-emphasis impulse response.

and

\[ s_I(t) = \sum_k \sqrt{2P_{I,k}} b^k(t - \tau_k) c^{(k)}(t - \tau_k) \cos(\omega_{I,k} + \theta_k) \]

This summation represents the WLL spread-spectrum DS-CDMA signals of the users, which are also interferers. The transmission used is a spread-spectrum DS-CDMA system via the base station, and the system affected is an FS-ML where \( b^k(t) \) is the modulating signal, \( c^{(k)}(t) \) is the pseudorandom sequence, and \( \tau_k \) are time delays.

**a) Interference Noise at the FDM/FM Receiver Output**

It is shown in [19] that the combined signal to enter the demodulator (limiter/discriminator) of the FS-ML system, which is affected by the WLL users, is given by
$s(t) = \text{Re}\, E\left\{ \sqrt{2P_0} \exp[j\omega_0 t + j\varphi(t)] \left[ 1 + \sum_k I^{(k)}(t) \exp[-j\varphi(t) + j\theta_n] \right] \right\}$

$= \text{Re}\, E\left\{ \sqrt{2P_0} A(t) \exp[j\omega_0 t + j\varphi(t) + j\lambda(t)] \right\}$ \hspace{1cm} (5.149)

where

$I^{(k)}(t) = \sqrt{\frac{P_{L,k}}{P_0}} \left[ b^{(n)}(t - \tau_n) c^{(n)}(t - \tau_n) \exp(j\omega_{\Delta} t) \right] \otimes h_{IF}(t)$ \hspace{1cm} (5.150)

and

$\lambda(t) = \text{Im}\left\{ \ln\left[ 1 + \sum_{k=1}^K I^{(k)}(t) \exp[-j\varphi(t) + j\theta_n] \right] \right\}$ \hspace{1cm} (5.151)

$f_{\Delta} = \frac{\omega_1 - \omega_0}{2\pi} = \frac{\omega_{\Delta}}{2\pi}$

At the demodulator output we get the signal

$u_D(t) = \frac{1}{2\pi} \frac{d}{dt} \left[ \varphi(t) + \lambda(t) \right]$ \hspace{1cm} (5.152)

where the first component represents the desired signal and the second one is the interference noise. Under the assumption that in real working conditions, the following restriction is valid:

$\left| \sum_{n=1}^N I^{(n)}(t) \right|_{\text{max}} < 1$ \hspace{1cm} (5.153)

and the expression for the interference noise at the limiter-discriminator (L-D) output could be written in the following form:
\[ \lambda(t) = \text{Im} \left\{ \sum_{m=1}^{\infty} \frac{(-1)^{m-1}}{m} \left\{ \sum_{k=1}^{K} f^{(k)}(t) \exp[-j\varphi(t) + j\theta_n] \right\}^m \right\} \] (5.154)

Similar expressions have been derived in Section 5.3.1. By applying the multinomial theorem for the autocorrelation function of the interference noise \( R_l(t) = \mathbb{E}\{l(t)l^*(t + \tau)\} \) (see Appendix A), we find:

\[ R_l(t) = \sum_{m=1}^{\infty} \frac{1}{4m^2} \sum_{m_1 + \ldots + m_N = N} \left( \frac{m!}{m_1! \ldots m_N!} \right)^2 \left[ \prod_{n=1}^{m_n} R_1^{m_n}(\tau) R_0^{m_n}(\tau)^* + \prod_{n=1}^{m_n} R_I^{m_n}(\tau) R_0^{m_n}(\tau) \right] \] (5.155)

where

\[ R_I^{m_n}(\tau) = \mathbb{E}\{\exp[jm_n\varphi(t) - jm_n\varphi(t + \tau)]\} \] (5.156)

and

\[ R_I^{m_n}(\tau) = \mathbb{E}\{[I^{(n)}(t)I^{(n)}(t + \tau)^*]^{m_n}\} \] (5.157)

Taking into account (5.153), for the autocorrelation function of the interference noise, \( R_l(t) = \mathbb{E}\{\lambda(t)\lambda^*(t + \tau)\} \), we have [19]:

\[ R_l(t) \equiv \frac{1}{8} \text{Re} \left\{ \sum_{n=1}^{N} \mathbb{E}\{R_I^{(m_n=1)}(\tau) R_0^{(m_n=1)}(\tau)^*\} \right\} \] (5.158)

By applying the Wiener-Khintchine theorem to the autocorrelation of \( \lambda(t) \), and taking into account (5.153), the interference noise PSD at the FDM-FM receiver output is

\[
S_{IN}(f) \equiv \frac{f^2}{4|H_P(jf)|^2} \frac{P_{l,k}}{P_0} \left\{ [S_{CDMA}(f-f_\Delta)|H_{IF}(jf)|^2 \otimes S_{FM}(-f')] + [S_{CDMA}(-f-f_\Delta)|H_{IF}(jf)|^2 \otimes S_{FM}(f)] \right\} \] (5.159)
where $S_{CDMA}(f)$ is the Fourier transform of the auto correlation of interfering CDMA signal.

The interference noise power in a telephony channel of the FS-ML system centered at $f_{ch}$ may be written as

$$N_I = \frac{2bS_{IN}(f_{ch})}{(\Delta f_0)^2} \text{ (mWp)}$$

(5.160)

where $b = 1.7$ kHz is the telephone channel psophometric band and the units of noise are in milliwatts in that band phorphometrically weighted. $\Delta f_0$ is the FM signal test tone deviation.

**b) Probability of Error at the DML Receiver Output**

In the case that the main telephone link was also digital (CDMA), we observe from the analysis so far that $C/I, S/I,$ and probability of error or BER are powerful design tools. In addition to designing wireless systems of acceptable performance, lately they have been used as quality measures [16] and as measures for optimization of other aspects of wireless systems, such as resource allocations and more specifically channel assignment. In the next chapter, we shall encounter the metrics to be used as a reference level in our effort to develop algorithms, which will lead to interference suppression.

For evaluating the interference effects on FS-DML due to the WLL, we used the formulas for the probability of error in AWGN, because the CDMA interference was considered white noise. Generally, this is a questionable approximation, but the relatively flat PSD of CDMA signal in the bandwidth of interest gives us a certain degree of confidence in the interference analysis.

If we denote the PSD of total CDMA signal by

$$N_I = \frac{P_{I,k}}{B_d} \int_{f_\Delta-B_d/2}^{f_\Delta+B_d/2} S_{CDMA}(f) \, df$$

(5.161)

where $B_d$ is the bandwidth of digital microwave link (DML) signal, BER for M-QAM, at the output of DML receiver are given by [19]:
\[ P_{e,M-QAM} \equiv \left( \frac{1}{ld(M)} \right) \left\{ 1 - \left( 1 - \frac{1}{\sqrt{M}} \right) \frac{1}{2} \text{erfc} \left( \sqrt{\frac{3}{2(M-1)}} \cdot \frac{E_{b,DML}ld(M)}{N_0 + N_I} \right) \right\}^2 \]

where \( E_{b,DML} \) is the mean energy per bit of DML signal.

References


