Chapter 2
Optical OFDM Basics

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2.1 Introduction

We have witnessed a dramatic increase of interest in orthogonal frequency-division multiplexing (OFDM) from optical communication community in recent years. The number of publications on optical OFDM has grown dramatically since it was proposed as an attractive modulation format for long-haul transmission either in coherent detection [1] or in direct detection [2, 3]. Over the last few years, net transmission data rates grew at a factor of 10 per year at the experimental level. To date, experimental demonstration of up to 1 Tb s$^{-1}$ transmission in a single channel [4, 5] and 10.8 Tb s$^{-1}$ transmission based on optical FFT have been accomplished [6], whereas the demonstration of real-time optical OFDM with digital signal processing (DSP) has surpassed 10 Gb s$^{-1}$ [7]. These progresses may eventually lead to realization of commercial transmission products based on optical OFDM in the future, with the potential benefits of high spectral efficiency and flexible network design.

This chapter intends to give a brief introduction on optical OFDM, from its fundamental mathematical concepts to the up-to-date experimental results. This is organized into seven sections, including this introduction as Sect. 2.1. Section 2.2 reviews the historical developments of OFDM and its application in
optical transmission. Section 2.3 describes the fundamentals and different flavors of optical OFDM. As this book focuses on optical nonlinearity, which is a major concern for long-haul transmission, the coherent optical OFDM (CO-OFDM) is mainly considered in this chapter. Section 2.4 gives an introduction on CO-OFDM. The procedures of the DSP are also discussed in detail in this section. Some promising research directions for CO-OFDM are presented in Sect. 2.5. Section 2.6 gives the summary of the chapter.

2.2 Historical Perspective of OFDM

OFDM plays a significant role in the modern telecommunications for both wireless and wired communications. The history of frequency-division multiplexing (FDM) began in 1870s when the telegraph was used to carry information through multiple channels [8]. The fundamental principle of orthogonal FDM was proposed by Chang [9] as a way to overlap multiple channel spectra within limited bandwidth without interference, taking consideration of the effects of both filter and channel characteristics. Since then, many researchers have investigated and refined the technique over the years and it has been successfully adopted in many standards. Table 2.1 shows some of the key milestones of the OFDM technique in radiofrequency (RF) domain.

Although OFDM has been studied in RF domain for over four decades, the research on OFDM in optical communication began only in the late 1990s [13]. The fundamental advantages of OFDM in an optical channel were first disclosed in [14]. In the late 2000s, long-haul transmission by optical OFDM has been investigated by a few groups. Two major research directions appeared, direct-detection optical OFDM (DDO-OFDM) [2,3] looking into a simple realization based on low-cost optical components and CO-OFDM [1] aiming to achieve high spectral efficiency and receiver sensitivity. Since then, the interest in optical OFDM has increased dramatically. In 2007, the world’s first CO-OFDM experiment with line rate of 8 Gb s$^{-1}$ was reported [15]. In the last few years, the transmission capacity continued to grow.

<table>
<thead>
<tr>
<th>Year</th>
<th>Event</th>
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<tbody>
<tr>
<td>1966</td>
<td>R. Chang, foundation work on OFDM [9]</td>
</tr>
<tr>
<td>1971</td>
<td>S.B. Weinstein and P.M. Ebert, DFT implementation of OFDM [10]</td>
</tr>
<tr>
<td>1985</td>
<td>L. Cimini, OFDM for mobile communications [12]</td>
</tr>
<tr>
<td>1995</td>
<td>DSL formally adopted discrete multi-tone (DMT), a variation of OFDM</td>
</tr>
<tr>
<td>1999 (2002)</td>
<td>Wireless LAN standard, 802.11 a (g), Wi-Fi</td>
</tr>
<tr>
<td>2004</td>
<td>Wireless MAN standard, 802.16, WiMax</td>
</tr>
<tr>
<td>2009</td>
<td>Long time evolution (LTE), 4 G mobile standard</td>
</tr>
</tbody>
</table>
Table 2.2  Progress of optical OFDM
1996  Pan and Green, OFDM for CATV [13]
2001  You and Kahn, OFDM in direct modulation (DD) systems [16]
      Dixon et al., OFDM over multimode fiber [14]
2005  Jolley et al., experiment of 10 Gb s$^{-1}$ optical OFDM over multimode fiber (MMF) [17]
      Lowery and Armstrong, power-efficient optical OFDM in DD systems [18]
2006  Lowery and Armstrong [2], and Djordjevic and Vasic [3], long-haul direct-detection optical OFDM (DDO-OFDM)
      Shieh and Athaudage, long-haul coherent optical OFDM (CO-OFDM) [15]
2007  Shieh et al. [15], 8 Gb s$^{-1}$ CO-OFDM transmission over 1,000 km
2008  Yang et al. [19], Jansen et al. [20], Yamada et al. [21], >100 Gb s$^{-1}$ per single channel CO-OFDM transmission over 1,000 km
2009  Ma et al. [4], Dischler et al. [5], Chandrasekhar et al. [22], >1 Tb s$^{-1}$ CO-OFDM long-haul transmission

about ten times per year. In 2009, up to 1 Tb s$^{-1}$ optical OFDM was successfully demonstrated [4, 5]. Table 2.2 shows the development of optical OFDM in the last two decades.

Besides offline DSP, from 2009 onward, a few research groups started to investigate real-time optical OFDM transmission. The first real-time optical OFDM demonstration took place in 2009 [23], 3 years later than real-time single-carrier coherent optical reception [24, 25]. The pace of real-time OFDM development is fast, with the net rate crossing 10 Gb s$^{-1}$ within 1 year [7]. Moreover, by using orthogonal-band-multiplexing (OBM), which is a key advantage for OFDM, up to 56 Gb s$^{-1}$ [26] and 110-Gb s$^{-1}$ [27] over 600-km standard signal mode fiber (SSMF) was successfully demonstrated. Most recently, 41.25 Gb s$^{-1}$ per single-band was reported in [28]. As evidenced by the commercialization of single-carrier coherent optical receivers, it is foreseeable that real-time optical OFDM transmission with much higher net rate will materialize in the near future based on state-of-the-art ASIC design.

2.3  OFDM Fundamentals

Before moving onto the description of optical OFDM transmission, we will review some fundamental concepts and basic mathematic expressions of OFDM. It is well known that OFDM is a special class of multi-carrier modulation (MCM), a generic implementation of which is depicted in Fig. 2.1. The structure of a complex multiplier (IQ modulator/demodulator), which is commonly used in MCM systems, is also shown at the bottom of the Fig. 2.1. The key distinction of OFDM from general multicarrier transmission is the use of orthogonality between the individual subcarriers.
2.3.1 Orthogonality Between OFDM Subcarriers and Subbands

The MCM transmitted signal $s(t)$ is represented as

$$s(t) = \sum_{i=-\infty}^{+\infty} \sum_{k=1}^{N_{sc}} c_{ki} s_k (t - i T_s)$$  \hspace{1cm} (2.1)$$

$$s_k (t) = \Pi(t) e^{j2\pi f_k t}$$  \hspace{1cm} (2.2)$$

$$\Pi(t) = \begin{cases} 1, & (0 < t \leq T_s) \\ 0, & (t \leq 0, t > T_s) \end{cases}$$  \hspace{1cm} (2.3)$$

where $c_{ki}$ is the $i$th information symbol at the $k$th subcarrier, $s_k$ is the waveform for the $k$th subcarrier, $N_{sc}$ is the number of subcarriers, $f_k$ is the frequency of the subcarrier, and $T_s$ is the symbol period, $\Pi(t)$ is the pulse shaping function. The optimum detector for each subcarrier could use a filter that matches the subcarrier waveform, or a correlator matched with the subcarrier as shown in Fig. 2.1. Therefore, the detected information symbol $c'_{ki}$ at the output of the correlator is given by

$$c'_{ki} = \frac{1}{T_s} \int_{0}^{T_s} r(t - i T_s) s_k^* \, dt = \frac{1}{T_s} \int_{0}^{T_s} r(t - i T_s) e^{-j2\pi f_k t} \, dt,$$  \hspace{1cm} (2.4)$$

where $r(t)$ is the received time-domain signal. The classical MCM uses nonoverlapped band-limited signals, and can be implemented with a bank of large number
of oscillators and filters at both transmit and receive ends [29, 30]. The major disadvantage of MCM is that it requires excessive bandwidth. This is because in order to design the filters and oscillators cost-effectively, the channel spacing has to be multiple of the symbol rate, greatly reducing the spectral efficiency. A novel approach called OFDM was investigated by employing overlapped yet orthogonal signal set [9]. This orthogonality originates from straightforward correlation between any two subcarriers, given by

$$\delta_{kl} = \frac{1}{T_s} \int_0^{T_s} s_k s^*_l \, dt = \frac{1}{T_s} \int_0^{T_s} \exp(j2\pi(f_k - f_l) t) \, dt$$

$$= \exp(j\pi(f_k - f_l) T_s) \frac{\sin(\pi(f_k - f_l) T_s)}{\pi(f_k - f_l) T_s}. \quad (2.5)$$

It can be seen that if the following condition

$$f_k - f_l = m \frac{1}{T_s} \quad (2.6)$$

is satisfied, then the two subcarriers are orthogonal to each other. This signifies that these orthogonal subcarrier sets, with their frequencies spaced at multiple of inverse of the symbol rate can be recovered with the matched filters in (2.5) without intercarrier interference (ICI), in spite of strong signal spectral overlapping. Moreover, the concept of this orthogonality can be extended to combine multiple OFDM bands into a signal with much larger spectral width. Such approach was first introduced in [19, 31] to flexibly expand the capacity of a single wavelength. This method of subdividing OFDM spectrum into multiple orthogonal bands is so-called “orthogonal-band-multiplexed OFDM” (OBM-OFDM).

Figure 2.2 shows the concept of orthogonal band multiplexing, where the entire spectrum is composed by \( N \) OFDM subbands. In order to maintain the orthogonality, the frequency spacing between two OFDM bands has to be a constant multiple of the subcarrier frequency spacing. The orthogonal condition between the different bands is given by \( \Delta f_G = m \Delta f \), where \( m \) is an integer. This guarantees that each OFDM band is an orthogonal extension of another, and is a powerful method to increase channel capacity by adding OFDM subbands to the spectrum.

![Fig. 2.2 Principle of orthogonal-band-multiplexed OFDM](image)
A schematic of the transmitter and receiver configuration for OBM-OFDM is shown in the Fig. 2.3. The method has been first proposed in [32], where it is called cross-channel OFDM (XC-OFDM). The unique advantage of this method is that the data rate can be simply extended or modified to specification in a bandwidth-efficient manner.

Upon reception, the spectrum can be divided into multiple subbands. The band-partitioning at the receiver is not necessary to be the same as the transmitter. Figure 2.4 shows an example of single-band detection and multiband detection. In the former case, the receiver local oscillator laser is tuned to the center of each band, and an anti-aliasing filter (Filter I) selects a single OFDM band to be detected separately. In the latter case, the received laser tuned to the center of the guard band, and an anti-aliasing filter (Filter II) separates two OFDM bands, which are converted into digital symbols and separated by further digital down-converters to be detected simultaneously. In either case, the inter-band interference (IBI) is avoided because of the orthogonality between the neighboring bands, despite the “leakage” of the subcarriers from neighboring bands. Thus, CO-OFDM can achieve high net rate by employing OBM without requiring DAC/ADC operating at extremely high sampling rates.
Fig. 2.5 Illustrations of three different methods used in [33] to detect a 1.2-Tb s$^{-1}$ 24-carrier NGI-CO-OFDM signal having 12.5-Gbaud PDM-QPSK carriers with 50-GS s$^{-1}$ ADC, (a) detecting 1 carrier per sampling with an oversampling factor of 4, (b) detecting 2 carriers per sampling with an oversampling factor of 2, and (c) detecting 3 carriers per sampling with an oversampling factor of 1.33. OLO Optical local oscillator

An additional advantage of the multi-band detection is its capability to save the number of required optical components at the receiver. One experimental demonstration of this has been shown in [33], where 24 orthogonal bands of OFDM are transmitted to generate a total of 1.2 Tb s$^{-1}$ data rate. In the receiver, three schemes are used: (1) detecting 1 band per ADC with an oversampling factor of 4, (2) detecting 2 bands per ADC with an oversampling factor of 2, and (3) detecting 3 bands per ADC with an oversampling factor of 1.33. All three schemes can recover the received signal completely. Assuming the ADC bandwidth is sufficiently wide, the more the number of bands are detected simultaneously, the less the number of the optical receivers are required (Fig. 2.5).

As mentioned earlier, the orthogonality condition is satisfied when the guard band $\Delta f_G$ is multiple of subcarrier spacing $\Delta f$. A generalized study of the influence of guard band to the system performance is shown in [34]. The validity of the orthogonality condition that minimizes the IBI was verified through experiment. Due to the IBI, the subcarriers at the edges of each band bear the largest inter-band penalty. Figure 2.6a, b show the received SNR of the “edge subcarriers” (the first and the last subcarrier of the band) as a function of the guard band normalized to the subcarrier spacing, at back-to-back and 1,000-km transmission, respectively. For simplicity, only one polarization is presented. The SNR oscillates as the guard spacing increases with a step size of half of the subcarrier spacing. It is shown in theory that ICI interference due to frequency spacing is a sinc function [35]. The SNR oscillation eventually stabilizes to a constant value, where effect of neighboring band can be considered negligible. By comparing with the stabilized SNR, the system penalty as a function of the guard band can be investigated. At 1,000 km transmission, when the guard band equals to a multiple of the subcarrier spacing, the SNR stabilizes at around a 10.5 dB, and the penalty almost decreases to zero, validating the assumption that guard band can be minimized for higher spectral efficiency using the orthogonal band multiplexing condition.
2.3.2 Discrete Fourier Transform Implementation of OFDM

We rewrite the expression of (2.1)–(2.3) for one OFDM symbol as:

\[
\tilde{s}(t) = \sum_{i=0}^{N-1} A_i \exp \left( j 2\pi \frac{i}{T} t \right), \quad 0 \leq t \leq T, \quad (2.7)
\]

which is the complex form of the OFDM baseband signal.

If we sample the complex signal with a sample rate of \( N/T \), and add a normalization factor \( 1/N \), then

\[
S_n = \frac{1}{N} \sum_{i=0}^{N-1} A_i \exp \left( j 2\pi \frac{i}{N} n \right), \quad n = 0, 1, \ldots, N - 1 \quad (2.8)
\]

where \( S_n \) is the \( n \)th time-domain sample. This is exactly the expression of inverse discrete Fourier transform (IDFT). It means that the OFDM baseband signal can be implemented by IDFT. The pre-coded signals are in the frequency domain, and
output of the IDFT is in the time domain. Similarly, at the receiver side, the data is recovered by discrete Fourier transform (DFT), which is given by:

\[
A_i = \sum_{i=0}^{N-1} R_n \exp \left(-j \frac{2\pi}{N} \frac{i}{n}\right), \quad n = 0, 1, \ldots, N - 1, \quad (2.9)
\]

where \(R_n\) is the received sampled signal, and \(A_i\) is received information symbol for the \(i\)th subcarrier. There are two fundamental advantages of DFT/IDFT implementation of OFDM. First, they can be implemented by (inverse) fast Fourier transform (I)FFT algorithm, where the number of complex multiplications is reduced from \(N^2\) to \(\frac{N^2}{2} \log_2 (N)\), slightly higher than linear scaling with the number of subcarriers, \(N\) [36]. Second, a large number of orthogonal subcarriers can be modulated and demodulated without resorting to very complex array of RF oscillators and filters. This leads to a relatively simple architecture for OFDM implementation when large number of subcarriers is required.

### 2.3.3 Cyclic Prefix for OFDM

In addition to modulation and demodulation of many orthogonal subcarriers via (I)FFT, one has to mitigate dispersive channel effects such as chromatic and polarization mode dispersions for good performance. In this respect, one of the enabling techniques for OFDM is the insertion of cyclic prefix [37, 38]. Let us first consider two consecutive OFDM symbols that undergo a dispersive channel with a delay spread of \(t_d\). For simplicity, each OFDM symbol includes only two subcarriers with the fast delay and slow delay spread at \(t_d\), represented by “fast subcarrier” and “slow subcarrier,” respectively. Figure 2.7a shows that inside each OFDM symbol, the two subcarriers, “fast subcarrier” and “slow subcarrier” are aligned upon the transmission. Figure 2.7b shows the same OFDM signals upon the reception, where the “slow subcarrier” is delayed by \(t_d\) against the “fast subcarrier.” We select a DFT window containing a complete OFDM symbol for the “fast subcarrier.” It is apparent that due to the channel dispersion, the “slow subcarrier” has crossed the symbol boundary leading to the interference between neighboring OFDM symbols, formally, the so-called inter-symbol-interference (ISI). Furthermore, because the OFDM waveform in the DFT window for “slow subcarrier” is incomplete, the critical orthogonality condition for the subcarriers is lost, resulting in an inter-carrier-interference (ICI) penalty.

Cyclic prefix was proposed to resolve the channel dispersion-induced ISI and ICI [37]. Figure 2.7c shows insertion of a cyclic prefix by cyclic extension of the OFDM waveform into the guard interval \(\Delta_G\). As shown in Fig. 2.7c, the waveform in the guard interval is essentially an identical copy of that in the DFT window, with time-shifted by “\(t_s\)” forward. Figure 2.7d shows the OFDM signal with the guard interval upon reception. Let us assume that the signal has traversed the same dispersive channel, and the same DFT window is selected containing a complete
OFDM symbol for the “fast subcarrier” waveform. It can be seen from Fig. 2.7d, a complete OFDM symbol for “slow subcarrier” is also maintained in the DFT window, because a proportion of the cyclic prefix has moved into the DFT window to replace the identical part that has shifted out. As such, the OFDM symbol for “slow
subcarrier” is an “almost” identical copy of the transmitted waveform with an additional phase shift. This phase shift is dealt with through channel estimation and will be subsequently removed for symbol decision. The important condition for ISI-free OFDM transmission is given by:

$$t_d < \Delta G.$$  \hspace{1cm} (2.10)

It can be seen that after insertion of the guard interval greater than the delay spread, two critical procedures must be carried out to recover the OFDM information symbol properly, namely, (1) selection of an appropriate DFT window, called DFT window synchronization, and (2) estimation of the phase shift for each subcarrier, called channel estimation or subcarrier recovery. Both signal processing procedures are actively pursued research topics, and their references can be found in both books and journal papers [37, 38].

The corresponding time-domain OFDM symbol is illustrated in Fig. 2.8, which shows one complete OFDM symbol composed of observation period and cyclic prefix. The waveform within the observation period will be used to recover the frequency-domain information symbols.

### 2.3.4 Spectral Efficiency for Optical OFDM

In DDO-OFDM systems, the electrical field of optical signal is usually not a linear replica of the baseband signal, and it requires a frequency guard band between the main optical carrier and OFDM spectrum, reducing the spectral efficiency. The net optical spectral efficiency is dependent on the implementation details. We will turn our attention to the optical spectral efficiency for CO-OFDM systems. In OFDM systems, \(N_{sc}\) subcarriers are transmitted in every OFDM symbol period of \(T_s\). Thus, the total symbol rate \(R\) for OFDM systems is given by

$$R = \frac{N_{sc}}{T_s}.$$  \hspace{1cm} (2.11)
Figure 2.9a shows the spectrum of wavelength-division-multiplexed (WDM) CO-OFDM channels, and Fig. 2.9b shows the zoomed-in optical spectrum for each wavelength channel. We use the frequency of the first null of the outermost sub-carrier to denote the boundary of each wavelength channel. The OFDM bandwidth, $B_{OFDM}$, is thus given by

$$B_{OFDM} = \frac{2}{T_s} + \frac{N_{sc} - 1}{t_s}, \quad (2.12)$$

where $t_s$ is the observation period (see Fig. 2.8). Assuming a large number of sub-carriers used, the bandwidth efficiency of OFDM $\eta$ is found to be

$$\eta = 2 \frac{R}{B_{OFDM}} = 2\alpha, \quad \alpha = \frac{t_s}{T_s}. \quad (2.13)$$
The factor of 2 accounts for two polarizations in the fiber. Using a typical value of 8/9, we obtain the optical spectral efficiency factor $\eta$ of 1.8 Baud/Hz. The optical spectral efficiency gives $3.6 \text{ b s}^{-1} \text{ Hz}^{-1}$ if QPSK modulation is used for each sub-carrier. The spectral efficiency can be further improved by using higher-order QAM modulation [39, 40]. To practically implement CO-OFDM systems, the optical spectral efficiency will be reduced by needing a sufficient guard band between WDM channels taking account of laser frequency drift about 2 GHz. This guard band can be avoided by using orthogonality across the WDM channels, which has been discussed in Sect. 2.3.1.

### 2.3.5 Peak-to-Average Power Ratio for OFDM

High peak-to-average-power ratio (PAPR) has been cited as one of the drawbacks of OFDM modulation format. In the RF systems, the major problem resides in the power amplifiers at the transmitter end, where the amplifier gain will saturate at high input power. One of the ways to avoid the relatively “peaky” OFDM signal is to operate the power amplifier at the so-called heavy “back-off” regime, where the signal power is much lower than the amplifier saturation power. Unfortunately, this requires an excess large saturation power for the power amplifier, which inevitably leads to low power efficiency. In the optical systems, interestingly enough, the optical power amplifier (predominately an Erbium-doped-amplifier today) is ideally linear regardless of its input signal power due to its slow response time in the order of millisecond. Nevertheless, the PAPR still poses a challenge for optical fiber communications due to the nonlinearity in the optical fiber [41–43].

The origin of high PAPR of an OFDM signal can be easily understood from its multicarrier nature. Because cyclic prefix is an advanced time-shifted copy of a part of the OFDM signal in the observation period (see Fig. 2.8), we focus on the waveform inside the observation period. The transmitted time-domain waveform for one OFDM symbol can be written as

$$s(t) = \sum_{k=1}^{N_{sc}} c_k e^{j2\pi f_k t}, \quad f_k = \frac{k - 1}{T_s}. \quad (2.14)$$

The PAPR of the OFDM signal is defined as

$$\text{PAPR} = \frac{\max \left\{ |s(t)|^2 \right\}}{E \left\{ |s(t)|^2 \right\}}, \quad t \in [0, T_s]. \quad (2.15)$$

For the simplicity, we assume that an M-PSK encoding is used, where $|c_k| = 1$. The theoretical maximum of PAPR is $10 \log_{10} \left( N_{sc} \right)$ in dB, by setting $c_k = 1$ and $t = 0$ in (2.14). For OFDM systems with 256 subcarriers, the theoretical maxim PAPR is
24 dB, which obviously is excessively high. Fortunately, such a high PAPR is a rare event such that we do not need to worry about it. A better way to characterize the PAPR is to use complementary cumulative distribution function (CCDF) of PAPR, \( P_c \), which is expressed as

\[
P_c = \Pr \{ \text{PAPR} > \zeta_p \},
\]

namely, \( P_c \) is the probability that PAPR exceeds a particular value of \( \zeta_p \).

Figure 2.10 shows CCDF with varying number of subcarriers. We have assumed QPSK encoding for each subcarrier. It can be seen that despite the theoretical maximum of PAPR is 24 dB for the 256-subcarrier OFDM systems, for the most interested probability regime, such as a CCDF of \( 10^{-3} \), the PAPR is around 11.3 dB, which is much less than the maximum value of 24 dB. A PAPR of 11.3 dB is still very high as it implies that the peak value is about one order of magnitude stronger than the average, and some form of PAPR reduction should be used. It is also interesting to note that the PAPR of an OFDM signal increases slightly as the number of subcarriers increases. For instance, the PAPR increases by about 1.6 dB when the subcarrier number increases from 32 to 256.

The sampled waveform is used for PAPR evaluation, and subsequently the sampled points may not include the true maximum value of the OFDM signal. Therefore, it is essential to oversample the OFDM signal to obtain accurate PAPR. Assume that over-sampling factor is \( h \), namely, number of the sampling points increases from \( N_{sc} \) to \( hN_{sc} \) with each sampling point given by

\[
t_l = \frac{(l - 1) T_s}{hN_{sc}}, \quad l = 1, 2, \ldots, hN_{sc}.
\]
Substituting \( f_k = \frac{k-1}{T_s} \) and (2.17) into (2.14), the \( l \)th sample of \( s(t) \) becomes

\[
s_l = s(t_l) = \sum_{k=1}^{N_{sc}} c_k e^ {j2\pi \frac{(k-1)(l-1)}{hN_{sc}}}, \quad l = 1, 2, \ldots hN_{sc}. \tag{2.18}
\]

Expanding the number of subcarriers \( c_k \) from \( N_{sc} \) to \( hN_{sc} \) by appending zeros to the original set, the new subcarrier symbol \( c'_k \) after the zero padding is formally given by

\[
c'_k = c_k, \quad k = 1, 2, \ldots, N_{sc}
\]

\[
c'_k = 0, \quad k = N_{sc} + 1, N_{sc} + 2, \ldots, hN_{sc}. \tag{2.19}
\]

Using the zero-padded new subcarrier set \( c'_k \), (2.18) is rewritten as

\[
s_l = \sum_{k=1}^{hN_{sc}} c'_k e^ {j2\pi \frac{(k-1)(l-1)}{hN_{sc}}} = F^{-1}(c'_k), \quad l = 1, 2, \ldots hN_{sc}. \tag{2.20}
\]

From (2.20), it follows that the \( h \) times oversampling can be achieved by IFFT of a new subcarrier set that zero-pads the original subcarrier set to \( h \) times of the original size.

Figure 2.11 shows the CCDF of PAPR varying oversampling factors from 1 to 8. It can be seen that the difference between the Nyquist sampling \((h = 1)\) and eight times oversampling is about 0.4 dB at the probability of \(10^{-3}\). However, most of the difference takes place below the oversampling factor of 4 and beyond this, PAPR changes very little. Therefore to use an oversampling factor of 4 for the purpose of PAPR, investigation seems to be sufficient.

![Fig. 2.11 Complementary cumulative distribution function (CCDF) for the PAPR of an OFDM signal with varying oversampling factors. The subcarrier number is fixed at 256](image)

**Fig. 2.11** Complementary cumulative distribution function (CCDF) for the PAPR of an OFDM signal with varying oversampling factors. The subcarrier number is fixed at 256
It is obvious that the PAPR of an OFDM signal is excessively high for either RF or optical systems. Consequently, PAPR reduction has been an intensely pursued field. Theoretically, for QPSK encoding, a PAPR smaller than 6 dB can be obtained with only a 4% redundancy [38]. Unfortunately, such code has not been identified so far. The PAPR reduction algorithms proposed so far allow for trade-off among three figure-of-merits of the OFDM signal: (1) PAPR, (2) bandwidth-efficiency, and (3) computational complexity. The most popular PAPR reduction approaches can be classified into two categories:

1. PAPR reduction with signal distortion. This is simply done by hard-clipping the OFDM signal [44–46]. The consequence of clipping is increased BER and out-of-band distortion. The out-of-band distortion can be mitigated through repeated filtering [46].

2. PAPR reduction without signal distortion. The idea behind this approach is to map the original waveform to a new set of waveforms that have a PAPR lower than the desirable value, most of the time, with some bandwidth reduction. Distortionless PAPR reduction algorithms include selective mapping (SLM) [47,48], optimization approaches such as partial transmit sequence (PTS) [49,50], and modified signal constellation or active constellation extension (ACE) [51,52].

### 2.3.6 Flavors of Optical OFDM

One of the major strengths of OFDM modulation format is its rich variation and ease of adaption to a wide range of applications. In wireless systems, OFDM has been incorporated in wireless LAN (IEEE 802.11a/g, or better known as WiFi), wireless WAN (IEEE 802.16e, or better known as WiMax), and digital radio/video systems (DAB/DVB) adopted in most parts of the world. In RF cable systems, OFDM has been incorporated in ADSL and VDSL broadband access through telephone copper wiring or power line. This rich variation has something to do with the intrinsic advantages of OFDM modulation including dispersion robustness, ease of dynamic channel estimation and mitigation, high spectral efficiency and capability of dynamic bit and power loading. Recent progress in optical OFDM is of no exception. We have witnessed many novel proposals and demonstrations of optical OFDM systems from different areas of the applications that aim to benefit from the aforementioned OFDM advantages. Despite the fact that OFDM has been extensively studied in the RF domain, it is rather surprising that the first report on optical OFDM in the open literature only appeared in 1998 by Pan et al. [13], where they presented in-depth performance analysis of hybrid AM/OFDM subcarrier-multiplexed (SCM) fiberoptic systems. The lack of interest in optical OFDM in the past is largely due to the fact the silicon signal processing power had not reached the point, where sophisticated OFDM signal processing can be performed in a CMOS integrated circuit (IC).

Optical OFDM are mainly classified into two main categories: coherent detection and direct detection according to their underlying techniques and applications. While direct detection has been the mainstay for optical communications over the
last two decades, the recent progress in forward-looking research has unmistakably pointed to the trend that the future of optical communications is the coherent detection.

DDO-OFDM has much more variants than the coherent counterpart. This mainly stems from the broader range of applications for direct-detection OFDM due to its lower cost. For instance, the first report of the DDO-OFDM \[13\] takes advantage of that the OFDM signal is more immune to the impulse clipping noise in the CATV network. Other example is the single-side-band (SSB)-OFDM, which has been recently proposed by Lowery et al. and Djordjevic et al. for long-haul transmission \[2, 3\]. Tang et al. have proposed an adaptively modulated optical OFDM (AMOOFDM) that uses bit and power loading showing promising results for both multimode fiber and short-reach SMF fiber link \[53, 54\]. The common feature for DDO-OFDM is of course using the direct detection at the receiver, but we classify the DDO-OFDM into two categories according to how optical OFDM signal is being generated: (1) linearly mapped DDO-OFDM (LM-DDO-OFDM), where the optical OFDM spectrum is a replica of baseband OFDM, and (2) nonlinearly mapped DDO-OFDM (NLM-DDO-OFDM), where the optical OFDM spectrum does not display a replica of baseband OFDM \[55\].

CO-OFDM represents the ultimate performance in receiver sensitivity, spectral efficiency, and robustness against polarization dispersion, but yet requires the highest complexity in transceiver design. In the open literature, CO-OFDM was first proposed by Shieh and Authaudage \[1\], and the concept of the coherent optical MIMO-OFDM was formalized by Shieh et al. in \[56\]. The early CO-OFDM experiments were carried out by Shieh et al. for a 1,000 km SSMF transmission at 8 Gb s\(^{-1}\) \[15\], and by Jansen et al. for 4,160 km SSMF transmission at 20 Gb s\(^{-1}\) \[57\]. Another interesting and important development is the proposal and demonstration of the no-guard interval CO-OFDM by Yamada et al. in \[58\], where optical OFDM is constructed using optical subcarriers without a need for the cyclic prefix. Nevertheless, the fundamental principle of CO-OFDM remain the same, which is to achieve high spectral efficiency by overlapping subcarrier spectrum yet avoiding the interference by using coherent detection and signal set orthogonality. As this book is primarily focused on fiber nonlinearity, coherent scheme will be mainly discussed in the following sections.

### 2.4 Coherent Optical OFDM Systems

Coherent optical communication was once intensively studied in late 1980s and early 1990s due to its high sensitivity \[59–61\]. However, with the invention of Erbium-doped fiber amplifiers (EDFAs), coherent optical communication has literally abandoned since the early of 1990s. Preamplified receivers using EDFA can achieve sensitivity within a few decibels of coherent receivers, thus making coherent detection less attractive, considering its enormous complexity. In the early twenty-first century, the impressive record-performance experimental demonstration using a differential-phase-shift-keying (DPSK) system \[62\], in spite of an incoherent form
of modulation by itself, reignited the interest in coherent communications. The second wave of research on coherent communications is highlighted by the remarkable theoretical and experimental demonstrations from various groups around the world [56, 63, 64]. It is rather instructive to point out that the circumstances and the underlying technologies for the current drive for coherent communications are entirely different from those of a decade ago, thanks to the rapid technological advancement within the past decade in various fields. First, current coherent detection systems are heavily entrenched in silicon-based DSP for high-speed signal phase estimation and channel equalization. Second, multicarrier technology, which has emerged and thrived in the RF domain during the past decade, has gradually encroached into the optical domain [65, 66]. Third, in contrast to the optical system that was dominated by a low-speed, point-to-point, and single-channel system a decade ago, modern optical communication systems have advanced to massive wave-division-multiplexed (WDM) and reconfigurable optical networks with a transmission speed approaching 100 Gb s\(^{-1}\). In a nutshell, the primary aim of coherent communications has shifted toward supporting these high-speed dynamic networks by simplifying the network installation, monitoring and maintenance.

When the modulation technique of OFDM combines with coherent detection, the benefits brought by these two powerful techniques are multifold [67]: (1) High spectral efficiency; (2) Robust to chromatic dispersion and polarization-mode dispersion; (3) High receiver sensitivity; (4) Dispersion Compensation Modules (DCM)-free operation; (5) Less DSP complexity; (6) Less oversampling factor; (7) More flexibility in spectral shaping and matched filtering.

### 2.4.1 Principle for CO-OFDM

Figure 2.12 shows the conceptual diagram of a typical coherent optical system setup. It contains five basic functional blocks: RF OFDM signal transmitter, RF to optical (RTO) up-converter, Fiber links, the optical to RF (OTR) down-converter, and the RF OFDM receiver. Such setup can be also used for single-carrier scheme, in which the DSP part in the transmitter and receiver needs to be modified, while all the hardware setup remains the same.

We will trace the signal flow end-to-end and illustrate each signal processing block. In the RF OFDM transmitter, the payload data is first split into multiple parallel branches. This is so-called “serial-to-parallel” conversion. The number of the multiple branches equals to the number of loaded subcarrier, including the pilot subcarriers. Then the converted signal is mapped onto various modulation formats, such as phase-shift keying (PSK), quadrature amplitude modulation (QAM), etc. The IDFT will convert the mapped signal from frequency domain into time domain. Two-dimensional complex signal is used to carry the information. The cyclic prefix is inserted to avoid channel dispersion. Digital-to-signal converters (DACs) are used to convert the time-domain digital signal to analog signal. A pair of electrical low-pass filters is used to remove the alias sideband signal. Figure 2.13 shows the effect of the anti-aliasing filter at the transmitter side.
At the RTO up-converter, the baseband OFDM $S_B(t)$ signal is upshifted onto optical domain using an optical I/Q modulator, which is comprised by two Mach–Zehnder modulators (MZMs) with a $90^\circ$ optical phase shifter. The up-converted OFDM signal in optical domain is given by

$$E(t) = \exp(j\omega_{LD1}t + \phi_{LD1})S_B(t),$$

(2.21)

where $\omega_{LD1}$ and $\phi_{LD1}$ are the frequency and phase of the transmitter laser, respectively. The optical signal $E(t)$ is launched into the optical fiber link, with an impulse response of $h(t)$. The received optical signal $E'(t)$ becomes

$$E'(t) = \exp(j\omega_{LD1}t + \phi_{LD1})S_B(t) \otimes h(t),$$

(2.22)

where $\otimes$ stands for the convolution operation.

When the optical signal is fed into the OTR converter, the optical signal $E'(t)$ is then mixed with a local laser at a frequency of $\omega_{LD2}$ and a phase of $\phi_{LD2}$. Assume the frequency and phase difference between transmit and receiver lasers are

$$\Delta \omega = \omega_{LD1} - \omega_{LD2}, \quad \Delta \phi = \phi_{LD1} - \phi_{LD2}$$

(2.23)
Then the received RF OFDM signal can be expressed as

\[ r(t) = \exp(j\Delta \omega t + \Delta \phi) S_B(t) \otimes h(t) \]  

(2.24)

In the RF OFDM receiver, the down-converted RF signal is first sampled by high speed analog-to-digital converter (ADC). The typical OFDM signal processing comprises five steps:

1. Window synchronization.
2. Frequency synchronization.
3. Discrete Fourier transform.
5. Phase noise estimation.

We here briefly describe the five DSP procedures [68]. Window synchronization aims to locate the beginning and end of an OFDM symbol correctly. One of the most popular methods was proposed by Schmidl and Cox [69] based on cross-correlation of detected symbols with a known pattern. A certain amount of frequency offset can be synchronized by a similar method, namely, the frequency offset can be estimated from the phase difference between two identical patterns with a known time offset. After window synchronization, OFDM signal is partitioned into blocks each containing a complete OFDM symbol. DFT is used to convert each block of OFDM signal from time domain to frequency domain. Then the channel and phase noise estimation are performed in the frequency domain using training symbols and pilot subcarriers, respectively. The details of these procedures are given in the following section. Note that the same procedures will also be followed for the real-time implementation.

2.4.2 OFDM Digital Signal Processing

2.4.2.1 Window Synchronization

The DSP begins with window synchronization in the OFDM reception. Its accuracy will influence the overall performance. Improper position of the DFT window on the OFDM signal will cause the inter-symbol interference (ISI) and ICI. In the worse case, the mis-synchronized symbol cannot be detected completely. The most commonly used method is Schmidl-Cox approach [69]. In this method, a preamble consisting of two identical patterns is inserted in the beginning of the multiple OFDM symbols, namely, an OFDM frame. Figure 2.14 shows the OFDM frame structure.

The Schmidl synchronization signal can be expressed as

\[ s_m = s_{m+Nsc/2}, \quad m = 1, 2, \ldots, Nsc/2. \]  

(2.25)
Considering the channel effect, from (2.24), the received samples will have the form as
\[ r_m = e^{j\Delta \omega t + \Delta \phi} s_m + n_m, \]  
(2.26)

where \( s_m = S_m(t) \otimes h(t) \). \( n_m \) stands for the random noise.

The delineation of OFDM symbol can be identified by studying the following correlation function defined as
\[ R_d = \sum_{m=1}^{N_{sc}/2} r_{m+d}^* r_{m+N_{sc}/2}. \]  
(2.27)

The principle is based on the fact that the second half of \( r_m \) is identical to the first half except for a phase shift. Assuming the frequency offset \( \omega_{\text{off}} \) is small to start with, we anticipate that when \( d = 0 \), the correlation function \( R_d \) reaches its maximum value.

### 2.4.2.2 Frequency Offset Synchronization

In wireless communications, numerous approaches to estimate the frequency offset between transmitter and receiver have been proposed. In CO-OFDM systems, we use the correlation from the window synchronization to obtain the frequency offset. The phase difference from the sample \( s_m \) to \( s_{m+N_{sc}/2} \) is \( \pi f_{\text{offset}} N_{sc}/S_{\text{ Sampling}} \), where \( S_{\text{ Sampling}} \) is the ADC sampling rate. The formula in Equation (2.27) can be re-written as
\[ R_d = \sum_{m=1}^{N_{sc}/2} |r_{m+d}|^2 e^{\pi f_{\text{offset}} N_{sc}/S_{\text{ Sampling}}}. \]  
(2.28)

Consequently, from the phase information of the correlation, the frequency offset can be derived as
\[ f_{\text{offset}} = \frac{S_{\text{ Sampling}}}{\pi N_{sc}} \angle R_d, \]  
(2.29)
where $\angle R_d$ stands for the angle of the correlation function of $R_d$. Because the phase information $\angle R_d$ ranges only from 0 to $2\pi$, large frequency offset cannot be identified uniquely. Thus, this approach only supports the frequency offset range from $-f_{sub}$ to $f_{sub}$ where $f_{sub}$ is the subcarrier spacing. To further increase the frequency offset compensation range, the synchronization symbol is further divided into $2^k (k > 1)$ segments [70]. The tolerable frequency offset can be enhanced to a few subcarrier spacing. Again, beside the Schmidl approach, there are other various approaches to perform the frequency offset estimation, such as the pilot-tone approach [71].

### 2.4.2.3 Channel Estimation

Assuming successful completion of window synchronization and frequency offset compensation, the RF OFDM signal after DFT operation is given by

$$r_{ki} = e^{j\phi_i}h_{ki}s_{ki} + n_{ki}, \quad (2.30)$$

where $s_{ki}$ ($r_{ki}$) is the transmitted (received) information symbol, $\phi_i$ is the OFDM common phase error (CPE), $h_{ki}$ is the frequency domain channel transfer function, and $n_{ki}$ is the noise. The common phase error is caused by the finite linewidth of the transmitter and receiver laser.

An OFDM frame usually contains a large number of OFDM symbols. Within each frame, the optical channel can be assumed to be invariant. There are various methods of channel estimation, such as time-domain pilot-assisted and the frequency-domain assisted approaches [3, 72]. Here, we are using the frequency domain pilot-symbol assisted approach. Figure 2.15 shows an OFDM frame in a time-frequency two-dimensional structure.

![Fig. 2.15 Data structure of an OFDM frame](image-url)
The first few symbols are the pilot-symbols or training symbols for which transmitted pattern is already known at the receiver side. The channel transfer function can be estimated as

$$h_{ki} = e^{-j\phi_i} r_{ki} / s_{ki}.$$  

Due to the presence of the random noise, the accuracy of the channel transfer function $h$ is limited. To increase the accuracy of channel estimation, multiple training symbols are used. By performing averaging over multiple training symbols, the influence of the random noise can be much reduced. However, training symbols also leads to increase of overhead or decrease of the spectral efficiency. In order to obtain accurate channel information while still using little overhead, interpolation or frequency domain averaging algorithm [73] over one training symbol can be used.

### 2.4.2.4 Phase Estimation

As we mentioned above, the phase noise is due to the linewidth of the transmitter and receiver lasers. For CO-OFDM, we assume that $N_p$ subcarriers are used as pilot subcarrier to estimate the phase noise. The maximum likelihood CPE is given as [68]

$$\phi_i = \arg \left( \sum_{k=1}^{N_p} r'_{ki} h^*_k s^*_k / \delta^2_k \right),$$

where $\delta_k$ is the standard deviation of the constellation spread for the $k$th subcarrier. After the phase noise estimation and compensation, the constellation for every subcarrier can be constructed and symbol decision is made to recover the transmitted data.

### 2.4.3 Polarization-Diversity Multiplexed OFDM

In Sect. 2.4.2, the OFDM signal is presented in a scalar model. However, it is well known that SSMF supports two modes in polarization domain. To describe the multiple input multiple output (MIMO) model for CO-OFDM mathematically, Jones vector is introduced and the channel model is thus given by [56]

$$s(t) = \sum_{i=-\infty}^{+\infty} \sum_{k=1}^{N_c} c_{ki} \Pi(t - i T_s) \exp(j 2\pi f_k (t - i T_s))$$

$$s(t) = \begin{bmatrix} s_x \\ s_y \end{bmatrix}, \quad c_{ik} = \begin{bmatrix} c_{x}^{ik} \\ c_{y}^{ik} \end{bmatrix}$$

$$f_k = \frac{k - 1}{T_s}$$

$$s_k(t) = \Pi(t) \exp(j 2\pi f_k t)$$
where $s_x$ and $s_y$ are the two polarization components for $s(t)$ in the time domain; $c_{ik}$ is the transmitted OFDM information symbol in the form of Jones vector for the $k$th subcarrier in the $i$th OFDM symbol; $c_{ik}^x$ and $c_{ik}^y$ are the two polarization components for $c_{ik}$; $f_k$ is the frequency for the $k$th subcarrier; $N_{sc}$ is the number of OFDM subcarriers; and $T_s$ and $t_s$ are the OFDM symbol period and observation period, respectively [56]. In [56] four CO-MIMO-OFDM configurations are described: (1) (1×1) single-input single-output, SISO-OFDM; (2) (1×2) single-input multiple-output SIMO-OFDM; (3) (2×1) multiple-input single-output MISO-OFDM; (4) (2×2) multiple-input multiple-output MIMO-OFDM. Among those configurations, SISO-OFDM and MIMO-OFDM are the preferred schemes. MIMO-OFDM is also called polarization diversity multiplexed (PDM) OFDM. Figure 2.16 shows the PDM-OFDM conceptual diagram.

In such a scheme, the OFDM signal is transmitted via both polarizations, doubling the channel capacity compared to the SISO scheme. At the receiver, no hardware polarization tracking is needed as the channel estimation can help the OFDM receiver to recover the transmitted OFDM signals on two polarizations.

Some milestone experimental demonstrations for CO-OFDM are given in Table 2.2. Among these proof-of-concept demonstrations, two milestones are especially attention-grabbing – OFDM transmission at 100-Gb s$^{-1}$ and 1-Tb s$^{-1}$. This is because 100 Gb s$^{-1}$ Ethernet has recently been ratified as an IEEE standard and increasingly becoming a commercial reality, whereas 1-Tb s$^{-1}$ Ethernet standard is anticipated to be available in the time frame as early as 2012–2013 [74]. In 2008, [19–21] demonstrated more than 100 Gb s$^{-1}$ over 1,000 km SSMF transmission. In 2009, [4, 5] showed more than 1 Tb s$^{-1}$ CO-OFDM transmission.

### 2.4.4 Real-Time Coherent Optical OFDM

The real-time optical OFDM has progressed rapidly in OFDM transmitter [75, 76], OFDM receiver [23, 26–28], and OFDM transceiver [7]. Because this chapter is focused on the long-haul transmission, we will mainly discuss the real-time CO-OFDM transmission in this subsection. With increased research interest in optical OFDM, numerous publications on this topic are being produced confirming the
fast pace of research. However, most of the published CO-OFDM experiments are based on off-line processing, which lags behind single-carrier counterpart, where a real-time transceiver operating at 40 Gb s\(^{-1}\) based on CMOS ASICs has already been reported [77]. More importantly, OFDM is based on symbol and frame structure, and the required DSP associated with OFDM procedures, such as window synchronization and channel estimation, remains a challenge for real-time implementation. Among many demonstrated algorithms, only a few can be practically realized due to various limitations associated with digital signal processor capability. It is thus essential to investigate efficient and realistic algorithms for real-time CO-OFDM implementation in both FPGA and ASIC platforms.

### 2.4.4.1 Real-Time Window Synchronization

The first DSP procedure for OFDM is symbol synchronization. Traditional offline processing uses the Schimdl approach [69], where the autocorrelation of two identical patterns inserted at the beginning of each OFDM frame gives rise to a peak indicating the starting position of the OFDM frame and symbol. The autocorrelation output is

\[
P(d) = \sum_{k=0}^{L-1} r_d^* r_{d+k} r_{d+k+L}.
\]  

(2.36)

and can be recursively expressed as

\[
P(d + 1) = P(d) + r_d^* r_{d+L} - r_d^* r_{d+L}.
\]  

(2.37)

An example of DSP implementation of (2.37) can be found in Fig. 2.17, where \(L\) indicates the length of synchronization pattern, \(r_d\) indicates the complex samples, and \(P(d)\) indicates the autocorrelation term whose amplitude gives peak when the synchronization is found. The relatively simple equation (2.37) and the architecture in Fig. 2.18, however, assume that the incoming signal is a serial stream, and this implementation only works if the process clock rate is the same as the sample rate.

![Fig. 2.17 DSP block diagram of autocorrelation for symbol synchronization based on serial processing](image)
This is because the moving window for autocorrelation needs to be taken sample by sample while multiple samples need to be processed simultaneously at a parallel process clock cycle. As there was no direct information available to indicate the frame starting point in the 16 parallel channels in our setup, locating the exact frame beginning would involve heavy computation that processes the data among all the channels. To illustrate this point, an implementation of the parallel autocorrelation can be constructed such that we can divide the autocorrelation of (2.36) by length $N$ for the $N$ parallel processing:

$$P(d) = \sum_{k=0}^{(L/N)N(k+1) - 1} \sum_{m=Nk}^{r_d^* m r_d + m + L},$$

(2.38)

which does not have an apparent recursive equation. The DSP realization is presented in Fig. 2.18. As shown in (2.38) and Fig. 2.18, by restricting the synchronization pattern length $L$ to multiple of the number of de-multiplexed bits $N$, a simple implementation of autocorrelation suitable for parallel processing is realized. However, for the case of $N = 16$ and $L = 32$, the processing resource required in this parallel implementation is estimated as 16 complex multipliers and $16 \times 15 + 16 = 256$ complex adders at each clock cycle. This indicates further efficiency improvement of symbol synchronization in parallel processing is desired.

### 2.4.4.2 Real-Time Frequency Offset Synchronization

Frequency offset between signal laser and local lasers must be estimated and compensated before further processing. The algorithm used in this stage is the same as (2.29). In the experiment, the local laser frequency is placed within $\pm 2$ subcarrier spacings from the signal laser, which guarantees that the phase difference $\phi$ between these two synchronization patterns remains bounded within $\pm \pi$. It can be
shown that the error of multiple of the subcarrier spacing has no significance. The frequency offset can be derived as:

\[ f_{\text{offset}} = \frac{\hat{\phi}}{(\pi T / 2)}. \] (2.39)

The COordinate-Rotation-DIGital-Computer (CORDIC) algorithm is used to calculate the frequency offset angle and compensate input data in vectoring and rotation modes, respectively. Figure 2.19 shows the frequency offset angle output against the sampling points with the frequency offset normalized to \( 2/(\pi T) \). Once the timing estimate signal from window synchronization stage is detected, the current output value of (2.39) is the correct frequency offset.

Once the frequency offset is obtained, frequency-offset compensation will be started. The implementation of frequency offset compensation in real-time is to use the cumulative phase information. The DSP diagram for frequency compensation is shown in Fig. 2.20. Assuming that \( \Delta \Phi \) is the phase difference between adjacent samples, which is derived from the auto-correlation, within one FPGA sampling period, \( N \) samples are distributed among the multiplexed channels. For the \( i \)th channel, the phase is cumulated as \( i \times \Delta \Phi \), and then compensated for that channel.

![Fig. 2.19](image1) Real-time measurement of frequency offset estimation for the OFDM signal. The frequency offset is normalized to \( 2/(\pi T) \)

![Fig. 2.20](image2) DSP diagram for frequency offset compensation
2.4.4.3 Real-Time Channel Estimation

Figure 2.21 shows the diagram for real-time CO-OFDM channel estimation. Once the OFDM window is synchronized, an internal timer will be started, which is used to distinguish the pilot symbols and payload. Two steps are involved in this procedure, channel matrix estimation and compensation. In the time slot for pilot symbols, the received signal is multiplied with locally stored transmitted pilot symbols to estimate the channel response. The transmitted pattern typically has very simple numerical orientation. Thus, multiplication can be changed into addition/subtraction of real and imaginary parts of the complex received signal, which can give additional resource saving. Taking average of the estimated channel matrixes over time and frequency can be used to alleviate error due to the random noise. Then the averaged channel estimation will be multiplied to the rest of the received payload symbols to compensate for the channel response. It is worth pointing out that one complex multiplier can be composed of only three (instead of four) real number multipliers.

To further save the hardware resources, the realization of the channel estimation can be done in a simple lookup table when pilot subcarriers are modulated with QPSK as in Table 2.3, avoiding the use of costly multipliers.

Fig. 2.21 Channel estimation diagram. *P.C.S* Pilot channel symbol; *C.E.S* Channel estimated symbol; *A.C.E.S* Averaged channel estimated symbol; *C.C.S* Compensated channel symbol
Table 2.3  Lookup table for channel and phase estimate in case of QPSK pilot subcarrier. Received signal is $R = a + jb$

<table>
<thead>
<tr>
<th>Message symbols of pilot</th>
<th>Modulated symbols of pilot</th>
<th>$H^{-1}$ or $B^{-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Real</td>
</tr>
<tr>
<td>0</td>
<td>$-1 + j$</td>
<td>$-a - b$</td>
</tr>
<tr>
<td>1</td>
<td>$-1 - j$</td>
<td>$-a + b$</td>
</tr>
<tr>
<td>2</td>
<td>$1 + j$</td>
<td>$a - b$</td>
</tr>
<tr>
<td>3</td>
<td>$1 - j$</td>
<td>$a + b$</td>
</tr>
</tbody>
</table>

Fig. 2.22  Phase estimation diagram

2.4.4.4  Real-Time Phase Estimation

Similar to channel estimation, phase estimation procedure can also be divided into estimation and compensation parts, which is shown in Fig. 2.22. Pilot subcarriers within one symbol will be selected by the inner timer. These pilot subcarriers then are compared with local stored transmitted pattern to obtain the phase noise information. The same symbol is delayed, and then compensated with the estimated phase noise factor.

2.4.5  Experimental Demonstrations for CO-OFDM, from 100 Gb s$^{-1}$ to 1 Tb s$^{-1}$, from Offline to Real-Time

Before 2008, the maximum line rate of CO-OFDM was limited to 52.5 Gb s$^{-1}$, insufficient to meet the requirement of 100 Gb s$^{-1}$ Ethernet. The main limitation is the electrical RF bandwidth of off-shelf DAC/ADC components. To implement 107 Gb s$^{-1}$ optical coherent OFDM based on QPSK, the required electrical
bandwidth is about 15 GHz. The best commercial DACs/ADCs in silicon IC at that time had a bandwidth of only 6 GHz [77], so the realization of 100 Gb s\(^{-1}\) CO-OFDM in a cost-effective manner remained challenging. To overcome this electrical bandwidth bottleneck associated with DAC/ADC devices, we used the orthogonal band multiplexing to demonstrate 107 Gb s\(^{-1}\) transmission over 1,000 km [19].

At the transmitter side, the 107 Gb s\(^{-1}\) OBM-OFDM signal is generated by multiplexing 5 OFDM subbands. In each band, 21.4 Gb s\(^{-1}\) OFDM signals are transmitted in both polarizations. The multi-frequency optical source with tones spaced at 6406.25 MHz is generated by cascading two intensity modulators (IMs). The guard-band equals to just one subcarrier spacing \((n = 1)\). The experimental setup for 107 Gb s\(^{-1}\) CO-OFDM is shown in Fig. 2.23. Figure 2.24 shows the multiple tones generated by this cascaded architecture using two IMs. Only the middle five tones with large and even power are used for performance evaluation. The transmitted signal is generated off-line by MATLAB program with a length of \(2^{15} - 1\) PRBS and mapped to 4-QAM constellation. The digital time domain signal is formed after IFFT operation. The FFT size of OFDM is 128, and guard interval is 1/8 of the symbol window. The middle 82 subcarriers out of 128 are filled, from which four pilot subcarriers are used for phase estimation. The I and Q components

![Experimental setup for 107 Gb s\(^{-1}\) OBM-OFDM systems](image)
of the time domain signal is uploaded onto a Tektronix Arbitrary Waveform Generator (AWG), which provides the analog signals at 10 GS s\(^{-1}\) for both I and Q parts. The AWG is phase locked to the synthesizer through 10 MHz reference. The optical I/Q modulator comprising two MZMs with 90° phase shift is used to directly impress the baseband OFDM signal onto five optical tones. The modulator is biased at null point to suppress the optical carrier completely and perform linear baseband-to-optical up-conversion [79]. The optical output of the I/Q modulator consists of five-band OBM-OFDM signals. Each band is filled with the same data at 10.7 Gb s\(^{-1}\) data rate and is consequently called “uniform filling” in this paper. To improve the spectrum efficiency, 2 × 2 MIMO-OFDM is employed, with the two OFDM transmitters being emulated by splitting the transmitted signal and recombining on orthogonal polarizations with a one OFDM symbol delay. These are then detected by two OFDM receivers, one for each polarization.

At the receiver side, the signal is coupled out of the recirculation loop and received with a polarization diversity coherent optical receiver [64, 80] comprising a polarization beam splitter, a local laser, two optical 90° hybrids, and four balanced photoreceivers. The complete OFDM spectrum comprises 5 subbands. The entire bandwidth for 107 Gb s\(^{-1}\) OFDM signal is only 32 GHz. The local laser is tuned to the center of each band, and the RF signals from the four balanced detectors are first passed through the anti-aliasing low-pass filters with a bandwidth of 3.8 GHz, such that only a small portion of the frequency components from other bands is passed through, which can be easily removed during OFDM signal processing. The performance of each band is measured independently. The detected RF signals are then sampled with a Tektronix Time Domain-sampling Scope (TDS) at 20 GS s\(^{-1}\). The sampled data is processed with a MATLAB program to perform 2×2 MIMO-OFDM processing.
Figure 2.25 shows the BER sensitivity performance for the entire 107 Gb s$^{-1}$ CO-OFDM signal at the back-to-back and 1,000-km transmission with the launch power of $-1$ dBm. The BER is counted across all five bands and two polarizations. It can be seen that the OSNR required for a BER of $10^{-3}$ is, respectively, 15.8 dB and 16.8 dB for back-to-back and 1,000-km transmission.

As 100-Gb s$^{-1}$ Ethernet has almost become a commercial reality, 1-Tb s$^{-1}$ transmission starts to receive growing attention. Some industry experts believe that the Tb/s Ethernet standard should be available in the time frame as early as 2012–2013 [74]. In the Tb/s experimental demonstrations [4, 5], we show that by using multiband structure of the proposed 1-Tb s$^{-1}$ signal, parallel coherent receivers each working at 30-Gb s$^{-1}$ can be used to detect 1-Tb s$^{-1}$ signal, namely, we have an option of receiver design in 30-Gb s$^{-1}$ granularity, a small fraction of the entire bandwidth of the wavelength channel. However, extension from current 100-Gb s$^{-1}$ demonstration to 1-Tb s$^{-1}$ requires tenfold bandwidth expansion, which is a significant challenge. To optically construct the multiband CO-OFDM signal using cascaded optical modulators, it entails ten times higher drive voltage, or use of the nonlinear fiber which may introduce unacceptable noise to the Tb/s signal. We here adopt a novel approach of multi-tone generation using a recirculating frequency shifter (RFS) architecture that generates 36 tones spaced at 8.9 GHz with only a single optical IQ modulator without a need for excessive high drive voltage. In this work, we extend the report of the first 1-Tb s$^{-1}$ CO-OFDM transmission with a record reach of 600 km over SSMF fiber and a spectral efficiency of 3.3 bit s$^{-1}$ Hz$^{-1}$ without either Raman amplification or optical compensation [81]. Our demonstration signifies that the CO-OFDM may potentially become an attractive candidate for future 1-Tb s$^{-1}$ Ethernet transport even with the installed fiber base.

Figure 2.26a shows the architecture of the RFS consisting of a closed fiber loop, an IQ modulator, and two optical amplifiers to compensate the frequency conversion loss. The IQ modulator is driven with two equal but 90° phase shifted RF tones through I and Q ports, to induce a frequency shifting to the input optical signal [82]. As shown in Fig. 2.26b, in the first round, an OFDM band at the center frequency of $f_1$ (called f1 band) is generated when the original OFDM band at the center frequency of $f_0$ passes through the optical IQ modulator and incurs a frequency shift equal to the drive voltage frequency of $f$. The $f_1$ band is split into two branches, one coupled out and the other recirculating back to the input of the optical IQ modulator.
In the second round, $f_2$ band is generated by shifting $f_1$ band along with a new $f_1$ band, which is shifted from original $f_0$ band. Similarly, in the $N$th round, we will have $f_N$ band shifted from the previous $f_{N-1}$ band, and $f_{N-1}$ shifted from previous $f_{N-2}$, etc. The $f_{N+1}$ band and beyond will be filtered out by the bandpass filter placed in the loop. With this scheme, the OFDM bands $f_1$ to $f_N$ are coming from different rounds and hence contain uncorrelated data pattern. In addition, such bandwidth expansion does not require excessive drive voltage for the optical modulator. Another major benefit of using the RFS is that we can adjust the delay of the recirculating loop to an integer number (30 in this experiment) of the OFDM symbol periods, and therefore the neighboring bands not only reside at the correct frequency grids, but are also synchronized in OFDM frame at the transmit. Replicating uncorrelated multiple OFDM bands using RFS is thus an extremely useful technique as it does not require duplication of the expensive test equipments including AWG and optical IQ modulators, etc. The RFS has been proposed and demonstrated for a tunable delay, but with only one tone being selected and used [82]. We here extend the application of RFS for multi-tone generation, or more precisely, for bandwidth expansion of uncorrelated multi-band OFDM signal.

Figure 2.27 shows the experimental setup for the 1-Tb s$^{-1}$ CO-OFDM systems. The optical sources for both transmitter and local oscillators are commercially available external-cavity lasers (ECLs), which have linewidth of about 100 kHz. The first OFDM band signal is generated by using a Tektronix AWG. The time domain OFDM waveform is generated with a MATLAB program with the parameters as follows: 128 total subcarriers; guard interval 1/8 of the observation period; middle
114 subcarriers filled out of 128, from which four pilot subcarriers are used for phase estimation. The real and imaginary parts of the OFDM waveforms are uploaded into the AWG operated at 10 GS/s to generate IQ analog signals, and subsequently fed into I and Q ports of an optical IQ modulator, respectively. The net data rate is 15 Gb/s after excluding the overhead of cyclic prefix, pilot tones, and unused middle two subcarriers. The optical output from the optical IQ modulator is fed into the RFS, replicated 36 times in a fashion described in Fig. 2.26b, and is subsequently expanded to a 36-band CO-OFDM signal with a data rate of 540 Gb/s. The optical OFDM signal from the RFS is then inserted into a polarization beam splitter, with one branch delayed by one OFDM symbol period (14.4 ns), and then recombined with a polarization beam combiner to emulate the polarization multiplexing, resulting in a net data rate of 1.08 Tb/s.

Figure 2.28a shows the multitone generation if the optical IQ modulation in Fig. 2.27 is bypassed. It shows a successful 36-tone generation with a tone-to-noise ratio (TNR) of larger than 20 dB at a resolution bandwidth of 0.02 nm. Figure 2.28b
shows the optical spectrum of 1.08 Tb s\(^{-1}\) CO-OFDM signal spanning 320.6 GHz in bandwidth consisting of 4,104 continuous spectrally overlapped subcarriers, implying a spectral efficiency of 3.3 bit s\(^{-1}\) Hz\(^{-1}\).

Figure 2.29 shows the BER sensitivity performance for the entire 1.08 Tb s\(^{-1}\) CO-OFDM signal at the back to back. The OSNR required for a BER of 10\(^{-3}\) is 27.0 dB, which is about 11.3 dB higher than 107 Gb s\(^{-1}\) we measured in [5]. The inset shows the typical constellation diagram for the detected CO-OFDM signal. The additional 1.3 dB OSNR penalty is attributed to the degraded TNR at the right-edge of the CO-OFDM signal spectrum (see Fig. 2.28a). Figure 2.30 shows the BER performance for all the 36 bands at the reach of 600 km with a launch power of 7.5 dBm, and it can be seen that all the bands can achieve a BER better than \(2 \times 10^{-3}\), the FEC threshold with 7% overhead. The inset shows the 1-Tb s\(^{-1}\) optical signal spectrum at 600-km transmission. It is noted that the reach performance for this first 1-Tb s\(^{-1}\) CO-OFDM transmission is limited by two factors: (1) the noise accumulation for

![Fig. 2.29 Back-to-back OSNR sensitivity for 1 Tb s\(^{-1}\) CO-OFDM signal](image)

![Fig. 2.30 BER performance for individual OFDM subbands at 600 km. The inset shows the optical spectrum of 1-Tb s\(^{-1}\) CO-OFDM signal after 600 km transmission](image)
the edge subcarriers that have gone through most of the frequency shifting, and (2) the two-stage amplifier exhibits over 9 dB noise figure because of the difficulty of tilt control in the recirculation loop. Both of the two issues can be overcome, and 1,000 km and beyond transmission at 1-Tb s\(^{-1}\) is practically reachable.

Another important development is the real-time CO-OFDM transmission. In 2009, 3.6 Gb s\(^{-1}\) per band CO-OFDM real-time OFDM reception was demonstrated by using a 54 Gb s\(^{-1}\) multi-band CO-OFDM signal [26]. Figure 2.31 shows the experimental setup and the DSP programming diagram of the real-time CO-OFDM receiver. At the transmitter, a data stream consisting of pseudo-random bit sequences (PRBSs) of length \(2^{15} - 1\) was first mapped onto three OFDM subbands with QPSK modulation. Three OFDM subbands were generated by an AWG at 10 GS s\(^{-1}\). Each subband contained 115 subcarriers modulated with QPSK. Two unfilled gap bands with 62 subcarrier-spacings were placed between the three subbands, which allowed them to be evenly distributed across the AWG output bandwidth. In each OFDM subband, the filled subcarriers, together with eight pilot subcarriers and 13 adjacent unfilled subcarriers, were converted to the time domain via inverse Fourier transform (IFFT) with size of 128. The number of filled subcarriers was restricted by the 1.2 GHz RF low-pass filter, which was used to select the subband to be received. A cyclic prefix of length 16 sample point was used, resulting in an OFDM symbol size of 144. The total number of OFDM symbols in each frame was 512. The first 16 symbols were used as training symbols for channel estimation. The real and imaginary parts of the OFDM symbol sequence were converted to analog waveforms via the AWG, before being amplified and used to drive an optical I/Q modulator that was biased at null. The transmitter laser and the receiver local laser were originated from the same ECL with 100-kHz linewidth through a 3-dB coupler. By doing so, frequency offset estimation was not needed in this experiment. The maximum net data rate of the signal after the optical modulation was 3.6 Gb s\(^{-1}\) for each OFDM subband. The multifrequency optical source contained 5 optical carriers at 9-GHz spacing, and was generated by using an MZM-driven by a high-power RF sinusoidal
wave at 9 GHz. The total number of subbands was then 15, resulting in a total net data rate of 54 Gb s\(^{-1}\). Unlike earlier works [19], the adjacent subbands in the multi-band OFDM signal contained independent data contents, more closely emulating an actual system. At the receiver, the OFDM signal in each sub-band was detected by a digital coherent receiver consisting of an optical hybrid and two single-ended input photodiode with a transimpedance amplifier (PIN-TIA). Two variable gain amplifiers (VGAs) amplified the signals to the optimum input amplitude before the ADCs, which were sampling at a rate of 2.5 GS s\(^{-1}\). The five most significant bits of each ADC were fed into an Altera Stratix II GX FPGA. All the CO-OFDM DSP was performed in the FPGA. The bit error rate was measured from the defined inner registers through embedded logic analyzer SignalTap II ports in Altera FPGA.

Figure 2.32 shows the measured BER as a function of optical signal-to-noise ratio (OSNR) for two cases: (1) a single 3.6-Gb s\(^{-1}\) CO-OFDM signal; (2) the center subband of the 54-Gb s\(^{-1}\) multi-band signal. In case (1), a BER better than \(1 \times 10^{-3}\) can be observed at OSNR of 3 dB. The OSNR is defined as the signal power in the subband under measurement over the noise power in a 0.1-nm bandwidth. In case (2), the required OSNR for BER \(1 \times 10^{-3}\) is 2.5 dB. There is virtually no penalty introduced by the band-multiplexing.

2.5 Promising Research Direction and Future Expectations

In this section, we consider some of the possible future research topics and trends of optical OFDM.

1. Optical OFDM for 1 Tb s\(^{-1}\) Ethernet transport.

As the 100 Gb s\(^{-1}\) Ethernet has increasingly become a commercial reality, the next pressing issue would be a migration path toward 1 Tb s\(^{-1}\) Ethernet transport to cope with ever-growing Internet traffic. In fact, some industry experts forecast that standardization of 1 TbE should be available in the time frame of 2012–2013 [74]. CO-OFDM may offer a promising alternative pathway toward Tb/s transport that possesses high spectral efficiency, resilience to
tributary timing misalignment and most important of all, the chromatic dispersion and PMD. Figure 2.33 shows the multiplexing and de-multiplexing architecture of CO-OFDM, where 1.2 Tb s\(^{-1}\) is divided into 12 frequency-domain tributaries at 100 Gb s\(^{-1}\) each. Using OBM scheme, OFDM can realize the high capacity without sacrificing spectral efficiency or increasing computational complexity [31].

2. MMF fiber for high spectral efficiency long-haul transmission.

MMF has long been perceived as a medium that is limited to short reach systems, although it can achieve very high capacity [83, 84]. Recent experiment of 20 Gb s\(^{-1}\) CO-OFDM transmission over 200 km MMF fiber may change that stereotype and spur research interests in MMF-based long-haul transmission [85]. The ideal MMF fiber for long-haul transmission may be the few-mode MMF fiber, for instance, dual-mode fiber, where space diversity can be utilized for MIMO gain. Figure 2.34 shows the conceptual diagram of an MMF-based
long-haul system. However, the MMF-based long-haul systems not only entail massive signal processing due to higher order MIMO reception, but it also requires many critical devices that are not employed in the conventional optical communications, such as the mode multiplexer and MMF amplifier.

3. Opto-electronic integrated circuits (OEICs) for optical OFDM
The notion of the OEICs that promises to place large number of the optical and electronic devices onto a single-chip can be traced back about four decades [86]. Because of the extensive signal processing involved in optical OFDM, it is natural to expect the silicon technology as the platform of choice to integrate the electronic DSPs and photonic components onto a single chip. Figure 2.35 shows a CO-OFDM transceiver architecture that includes four functional blocks including baseband OFDM transmitter, RF-to-optical (RTO) up-converter, optical-to-RF (OTR) down-converter, and baseband OFDM Receiver. We believe that the future advances in silicon OEIC will open up new venues for the coherent optical transmission technology and these will make inroads into the broad range of optical communication applications from access to core optical networks. We anticipate that the realization of optical OFDM subsystems/systems based on compact, power-efficient OEIC will present huge challenges and rich opportunities due to potential cost saving by law of scaling.

Fig. 2.35 Functional blocks of a CO-OFDM transceiver and its corresponding mapping to an integrated silicon chip
There are some other promising directions, such as adaptive coding in optical OFDM, optical OFDM-based access networks, standardizations, etc. Interested readers shall refer to [55] for more detailed discussion.

2.6 Conclusion

Optical OFDM transmission has become a fast progressing and vibrant research field in optical fiber communications. Last few years saw experimental demonstrations up to 1 Tbps transmissions, together with rapid advance in real-time demonstrations. With the standardization of 100 GbE and prospect of emergence of the Tb/s era, much excitement is growing in the optical communications community for the application of OFDM, the modulation format of choice in RF wireless communications. The introduction of OFDM without doubt has great potential and promise in bringing about the next-generation optical networks that possess high degree of flexibility and scalability. In the meantime, the research in optical OFDM also presents tremendous challenges and opportunities in the areas of novel DSP algorithms, high-speed electronic and photonic integrated circuits.

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