Quantifying the Risks of Trading

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Abstract

This article defines and describes methods for measuring three of the prominent risks of trading: valuation risk, market risk and counterparty credit risk. A fourth risk, operational risk, will not be discussed. The first section of the article describes the essential components of discounted cash flow models used for valuation, identifies the sources of valuation error and classifies the types of market factors needed to measure market value. The second section of the article describes the nature of and the methods that can be used to measure, monitor and limit market risk. A similar analysis of counterparty credit exposure and counterparty credit risk follows. Finally, the nature of and methods for measuring market risk and counterparty credit exposure will be compared and contrasted.

1 Introduction

The term ‘risk’ is used in finance in two different but related ways: as the magnitude of (a) the potential loss or (b) the standard deviation of the potential revenue (or income) of a trading or investment portfolio over some period of time.\(^1\)

Our discussion and analysis of market risk and counterparty credit risk will almost exclusively focus on risk as potential loss. That is, we will describe methods for measuring, in a specified context, the potential loss of economic value of a portfolio of financial contracts. The context that needs to be specified includes the time frame over which the losses might occur (e.g. a day, a year), the confidence level at which the potential loss will be measured (e.g. 95%, 99%) and the types of loss that would be attributed to the risk being measured (e.g. losses due to changes in market rates vs. losses due to...

\(^1\)The quantitative relationship between risk as potential loss and risk as uncertainty in future revenue is a function of the estimated probability distribution of future revenue. For example, if the estimated probability distribution of potential revenue is normally distributed around an expected value of zero then the potential loss at some confidence level can simply be expressed in terms of the standard deviation of potential revenue. In many cases the expected total revenue from a trading business is not zero (else the firm would not be in the business) and the probability distribution of future revenue may not be symmetric about its expected value.
the default of a counterparty). Part of the context for measuring the potential loss, whether due to market or credit risk, is the distinction between an economic perspective and an accounting perspective.

The distinction arises for market risk because the income from financial contracts may be accounted for in one of two ways: by accrual accounting (e.g. as is typically done for a portfolio of deposits and loans) or by mark to market accounting (e.g. as is typically done for a trading portfolio). The primary difference between the two approaches is in the timing of their recognition of financial gains or losses. Only the mark to market approach is equivalent to the continual measurement of economic value and change in economic value. The relative merits and demerits of measuring the income and risk of a particular business on an accrual basis will not be evaluated here. This article is focused on the risks of trading and will analyze and describe market risk from an economic perspective.

A similar issue arises for credit risk. One example of this is the potential difference between the loss in the market (economic) value of a loan caused by the default of the borrower and the timing of the recognition of the loss in the income statement.

A more important example of this issue for credit risk is the treatment of the deterioration of a borrower’s credit worthiness. Consider as an example a corporate loan. Assume that in the period after the loan was made the only relevant factor to undergo a material change was a deterioration of the credit worthiness of the borrower. In more detail, assume that one year ago a bank made a three year loan to a corporation for which the corporate borrower was required to make periodic interest payments and to pay back the principal and the remaining interest payment on the maturity date of the loan. Assume further that both the public credit rating and the bank’s internal credit rating of the borrower has deteriorated since the loan was initially made. Finally, assume that general market rates have remained unchanged since the loan was made and that the borrower has made all interest payments on time.

As a simple example of the difference consider two portfolios. Portfolio A is a standard deposit and loan portfolio. It consists of a ten year $100 million loan to firm X at a fixed semi-annual rate of 10.00% and a one year $100 million deposit from firm Y, at a semi-annual rate of 9.50% interest. Portfolio B is a trading portfolio. It consists of a long position in a ten-year debt security issued by firm X at a fixed semi-annual rate of 10.00% and a short position in a one-year debt security issued by firm Y at a fixed semi-annual rate of 9.50%. If both portfolios were viewed from a marked to market perspective, they would have identical market risks. However, under standard accounting practices, the effect of a change in market rates on the reported revenues of the portfolios will differ. Assume the only change in market rates is a 1% parallel increase in the risk free rate at all tenors. On a marked to market basis the net value of the securities in Portfolio B would fall in value. In contrast, the accrued interest earned by Portfolio A is locked in for the year and is independent of the level of interest rates. If interest rates should continue at their higher level the accrued interest earned by Portfolio A will only be affected after its one year deposit matures and has to be replaced with a deposit at a higher interest rate.
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Under standard accrual accounting the loan would be recorded on the balance sheet of the bank at its par value. The bank would only record a loss if the borrower defaulted on a payment. The bank would not in general recognize any loss due to the deterioration of the credit quality of the borrower. At most the bank could establish a general loan loss reserve for the expected credit loss of the portfolio.

In contrast, the market value of the loan would fall if the borrower’s credit worthiness deteriorated. To appreciate the reality of this loss, assume the bank took action to actively managing its credit risk to this corporate borrower, after the borrower’s credit risk had deteriorated. For example, if the bank were to sell the loan in the secondary loan market, then pari passu, it would suffer an economic loss – i.e. the market value of the loan would be less than its par amount because of the increased credit riskiness of the borrower. Or, if the bank tried to hedge its credit exposure to the borrower by buying a credit derivative on its underlying loan it might have to pay an annual fee for that credit insurance that was higher than the net interest income it was earning on the loan. Both of these examples of active portfolio management illustrate that a deterioration in the credit quality of the borrower, all other things held constant, will cause the market value of the loan to decrease, even if the borrower had not defaulted.

From the accrual accounting perspective no credit loss would occur without a default by the borrower. From the economic perspective, the increased riskiness of the borrower would cause the economic value of the loan to decrease.

This article will not focus on loan portfolio credit risk. It will however analyze another form of credit risk, the risk that the counterparty to a forward or derivative trade could default prior to the final settlement of the cash flows of the transaction. This form of credit risk is called counterparty pre-settlement credit risk and will be analyzed in detail below.

We will describe methods for measuring four aspects of the risks of trading:

- Methods for measuring and controlling valuation uncertainty and valuation error.
- Methods for measuring market risk. These methods measure the potential decrease in the economic value of contracts caused by potential future changes in market rates.
- Methods for measuring a counterparty’s pre-settlement credit exposure. These methods measure the potential future replacement cost of the forward and derivative contracts transacted with a counterparty, should the counterparty default at some time in the future before all the contracts mature. The potential credit exposure will depend on the potential future market value of the contracts transacted with the counterparty, on any risk mitigating agreements (such as netting) that have
been contracted with the counterparty and on the legal enforceability of such agreements.

- Methods for measuring counterparty credit risk. These methods measure the probability distribution of loss due to counterparty default and rest, in part, on measurements of the potential future credit exposure to the counterparty, the future default probability of the counterparty and the potential loss in the event of counterparty default.

Our measurements of market risk, counterparty credit exposure and counterparty credit risk all rest on our ability to measure the current and the potential future economic value of financial contracts. At the end of the article we will summarize and contrast each type of risk measurement. Because of the crucial connection between methods for valuing contracts and methods of risk measurement, we shall begin our discussion with a review of revaluation models, valuation errors and market factors.

2 Market valuation and valuation uncertainty

2.1 Discounted cash flow formula

Marking to market is the activity of ascertaining the market value of each financial instrument in a trading portfolio. Market value is ascertained in one of two ways: directly, by observing the market price of identical (or nearly identical) instruments or indirectly, by using a discounted cash flow revaluation model. When a discounted cash flow model is used, it is necessary to periodically calibrate the model against the market to ensure that the model’s valuation corresponds to the market’s.

Very liquid, cash-like financial instruments such as spot FX, equities and simple debt securities are marked to market by discovering the prices or rates at which identical (or nearly identical) instruments are traded in the market. For example, the market value of a portfolio of US Treasury securities of different maturities and coupon rates would be calculated simply by discovering the unit market price of each security in the portfolio and by multiplying the unit price by the number of units owned (positive for long, negative for short). Nothing beyond simple arithmetic would be needed to calculate the mark to market value of the portfolio.

In contrast, forward and derivative contracts are revalued in terms of discounted cash flow models (reval models). In essence, a reval model calculates the net present value of the expected future cash flows of the contract. It does this by representing the economic value of a contract as a function of its terms and conditions, basic market rates, and the current date:

$$PV(t)_k = f(T&C_k, \{X_j(t)\}, t),$$

(2.1)
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where:

\[ PV(t)_k \] is the present value of contract \( k \), at time \( t \);

\( T\&C_k \) are the terms and conditions of contract \( k \);

\( X_j(t) \) is the value of market factor \( j \), at time \( t \);

\( \{X_j(t)\} \) are the values of the complete set of market factors \( j \), at time \( t \), needed to value contract \( k \);

\( t \) is today’s date.

As an example, the terms and conditions of a simple interest rate swap would include: (a) a description of which party is paying the fixed cash flow and which is paying the floating; (b) the specification of the dates (and time of day) at which each floating rate will be set and the reference rate (e.g. three month US$ LIBOR) for setting the floating rate; (c) the specification of the dates at which fixed and floating cash flows will settle; and so forth.

Market factors \( \{X_j(t)\} \) are the fundamental market prices and rates in terms of which contracts are valued. Examples include the term structure of interest rates, spot FX rates, spot equity and equity index prices, spot and forward commodity prices and the term structures of implied volatilities of the preceding market factors.

For the simple interest rate swap, described above, the relevant market factors needed to value of the contract are the term structure of LIBOR interest rates. Given the terms and conditions of the interest rate swap, the current term structure of LIBOR interest rates and today’s date, the simple swap revaluation formula estimates the forward interest rate as of each floating rate reset date and then calculates the discounted present value of all the fixed and expected floating cash flows from their settlement date to today.

More generally, given the terms and conditions of a financial contract, the current values of the relevant market factors and today’s date, reval models estimate all the floating and contingent cash flows; discount all fixed, floating and contingent cash flows to present value; and convert cash flows denominated in different currencies into some base currency.

2.2 Types of revaluation models

There are two basic type of revaluation models: arbitrage-free, risk-neutral models and behavioral models. The most common type of financial model described in textbooks are the former. The essential feature of such models is their derivation by means of an arbitrage-free analysis. That is, an analysis which demonstrates that if the value of the contract were different from that given by the model it would be possible to risklessly make a return higher
than the risk-free rate by constructing a portfolio consisting of the contract and some other financial contracts with suitable weights.

Examples of risk-neutral, arbitrage-free models are the standard formula for valuing a forward FX contract in a liquid market or the Black–Scholes valuation formula for a European equity option.

The second basic class of models are behavioral models, such as those for mortgage-backed securities. Instruments that require behavioral models have some option-like feature (e.g. a prepayment option) whose exercise is only partly a function of the level of market rates (e.g. some people with mortgages may sell their home, and therefore prepay their mortgage, for reasons other than the level of interest rates). The likelihood that the pre-payment option will be exercised in the future can be approximately modeled as a function of the future level of interest rates on the basis of the historical patterns of the pre-payment behavior of borrowers. Another feature of behavioral models is that given the uncertainty in modeling the potential future cash flows as a function of interest rates, the expected value of the future cash flows are not discounted to present value at a risk-free rate (e.g. LIBOR). Instead, they are discounted to present value at some spread (‘option adjusted spread’) over LIBOR to take into account both the uncertainty in modeling the prepayment option and the uncertain liquidity of the market.

Examples of financial assets and liabilities that require behavioral models to value them include mortgages (and mortgage backed securities), credit card balances and demand deposit account balances.

When contracts are marked to market with a model, whether it is of risk-neutral arbitrage-free or behavioral type, the model ultimately function as complex interpolation tool.

2.3 Revaluation systems and valuation error

Revaluation models function within revaluation systems. To understand potential errors in valuation and to understand the steps of calculating market risk, we need to describe the essential components of a revaluation system.

3As an example of how a model functions as an interpolation tool, consider a portfolio of European FX options (i.e. simple puts and calls) on the US Dollar/Japanese Yen. The spot exchange rate and the yield curves of both currencies are easily observable. We can observe the prices of a set of options in the market and from these observed prices, using the standard FX option formula, we can infer the corresponding implied volatilities. For a sufficiently large set of observed option prices (i.e. for a sufficient range of strike prices and option tenors) we can construct an implied volatility surface from which any particular FX option could be valued. In effect, for each currency pair (e.g. US$/JPY), the model allows us to transform the observed prices of options that have standard strike prices and tenors into an implied volatility surface. The model then enables us to value an FX option on that currency pair, with any particular tenor and strike price (within a range), simply by interpolating in the implied vol space and calculating the corresponding value of the FX option.
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Table 1 and Figure 1 depict the essential components and flow of information in a revaluation system.

2.4 Sources of valuation differences and valuation error

A given contract might be valued differently by two different valuation systems. The difference in valuation may be large (as a percentage of total valuation) or small. There are several potential causes of valuation difference between systems: (a) the set of observed market factors chosen as input; (b) the set of transformation algorithms applied by each system to the observed market data; and (c) the revaluation algorithm as such.

For a given contract (or class of contracts) a valuation system might calculate a value that differs from the market’s valuation. This can arise for all the reasons that would cause two systems to calculate different valuations, as discussed in the previous paragraph. In addition, the market’s and the system’s valuation may differ because there may be causes or factors affecting the price in the market place that are not captured or incorporated in the revaluation model. That is why it is important to periodically calibrate the system’s valuation to the market’s.
Table 1. Primary Components of a Revaluation System

A Input of Observed Market Data and Other Parameters
- Market data are entered into templates formatted for the input of specific types of rates or prices.
- For some contracts, non-observable market factors are estimated by extrapolation or interpolation from observed market data (see below, under ‘Transformation of Market Data’). All algorithms that perform such extrapolation should be documented and periodically evaluated.

B Transformation of Market Data
- Observed market data will usually be transformed by algorithms:
  - Illiquid Prices. Sometimes a contract is transacted with unusual terms and conditions such that the market rates that are needed for its revaluation cannot be observed. For example, an FX option may be transacted at an unusually long tenor, beyond the longest expiration date for which option prices or option implied volatilities are readily observable in the market. When this occurs, the unobserved market data (e.g. the implied volatility of the long dated option) will be inferred by some extrapolation/interpolation algorithms from observed data.
  - Revaluation formulae require the calculation of zero coupon discount factors at specified forward dates. Depending on the type of contract, other intermediate variables, such as forward interest rates between specified pairs of forward dates, may also be needed for the valuation. These intermediate variables are derived from the observed and inferred market factors by transformation algorithms.

C Input of Contract Terms and Conditions
- The terms and conditions of each contract, for each form of contract, are entered via product specific templates and are stored in a suitably formatted database.

D Valuation Formula/Process
- Each contract is revalued according to the revaluation formula (or process) specific to the particular form of the contract, its detailed terms and conditions and the transformed market data.
- Factor sensitivities (which are critically important for measuring market risk; see below) are usually calculated as part of the revaluation process by revaluing each contract many times, under different scenarios in which some market rates are changed from its current level. For example, during the end-of-day batch process, each contract might be revalued $N + 1$ times. One revaluation will be at the current set of market factors. The other $N$ additional revaluations will be done, one for each of the $N$ factor sensitivity scenarios. These $N$ additional revaluations enable us to calculate $N$ factor sensitivities.
As an example of how valuation differences can occur even for a relatively simple product in a liquid market, consider a simple fixed/floating US$ LIBOR interest rate swap. The value of a simple, ‘plain vanilla’ US$ LIBOR swap of specified terms and conditions is determined once the appropriate set of LIBOR zero coupon discount factors (zcdfs) are specified.4 What observable market data are available to define a continuous set of US$ LIBOR zcdfs?

Information about US$ LIBOR interest rates appears in the market in several different forms: There are: (a) LIBOR money market rates from one to twelve months (e.g. the London Interbank Offer Rate at which banks offer to lend to each other); (b) market prices for about forty Eurodollar future contracts, one for each quarter for approximately the next ten years (a LIBOR future contract has a final value which is set on some specified future date and is determined by the then current three month LIBOR interest rate); and (c) LIBOR interest rate swap spreads over a range of tenors (e.g. the difference between the fixed rate of a US$ LIBOR swap and the US Treasury yield-to-maturity of the same tenor) and the US Treasury yield curve for a range of tenors (note that the current US Treasury yield, of a given tenor, plus the current swap spread, for that tenor, equals the current fixed rate of the swap, for that tenor). The different forms of market data overlap. Consequently there are choices in selecting the observed market data that will be used to derive the LIBOR zero coupon discount factors needed to value a general US$ LIBOR swap. For example, a particular valuation system might have as input: (a) US$ LIBOR money market rates for some specific set of tenors (e.g. overnight, 1 week, 1 month, 2 months, 3 months); (b) a set of three-month Eurodollar future contracts (e.g. the first sixteen contracts); (c) a set of US Treasury yields at specified maturities (e.g. two years, three years, five years to thirty years) and a set of corresponding US$ LIBOR swap spreads to Treasuries.

After the observed market data are selected, algorithms must be employed to transform them into a consistent set of zero coupon discount factors (zcdfs), one for each future date needed for valuation. There are several methods for making the transformation (e.g. the transformation could assume continuously compounded zero coupon rates or could assume constant forward rates; the transformation could make one of several different assumptions about the relationship between the price of a US$ LIBOR Eurodollar future and the corresponding US$ LIBOR forward rate, etc.). Consequently, the set of zcdfs used for valuation are not uniquely specified by the market because there are alternative sets of observed market data to choose from and there are alternative methods for transforming a given set of observed data into a set of zcdfs.

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Simple interest rate derivatives (e.g., interest rate swaps, simple interest rate caps or floors) have standard revaluation formula. Consequently, almost all of the differences in valuation of simple interest rate derivatives between different systems will be caused by differences in the choices of observed market data and differences in the transformation and interpolation algorithms used on the observed market data. The ultimate test of the validity of the entire valuation system is a comparison of the value obtained from the valuation system to the value observed in the market place. For the liquid US$ swap market the differences in valuation caused by different choices of input and different transformation algorithms, as a percentage of notional principal, should be relatively very small. In less liquid markets the difference in valuation may be quite large and material.

In summary, the market value calculated by a valuation system depends on: (a) the observed market factors that are selected; (b) the algorithms used to transform and interpolate the observed data into inferred market data and intermediate market rates (e.g. zedfs); and (c) the revaluation model or process itself. To the extent there is a reasonable range of choice in any of the components of valuation, the valuation system could generate a range of valuations. The magnitude of valuation uncertainty is called *valuation risk*. It is a misnomer to call this ‘model’ risk because doing so attributes the entire uncertainty of the valuation to the revaluation model and misdirects attention from the other components of the valuation process.

A valuation error occurs when there is a difference between the market’s valuation of a contract and the value assigned by a valuation system. In a liquid market, with small bid/offer spreads, the range of potential valuation is constrained (usually tightly constrained) by observable rates in the market.

To prevent or minimize valuation losses, a firm should take two actions:

- It should set aside a reserve for valuation uncertainty, whenever there is material uncertainty in the value assigned by the valuation system, given the choices or uncertainty in each step of the valuation process. Setting aside such a reserve requires careful scrutiny of accounting and regulatory standards.

- It should periodically calibrate the valuation system by comparing the system’s valuations to the market’s, to the degree that is possible, to ensure they are essentially the same.

My experience is that most of the large valuation losses that have been reported by firms in recent years can be attributed to either (a) errors in the values of the market factors used for valuation or (b) errors in the algorithms used to extrapolate from observed market rates to inferred, unobserved rates. Valuation losses caused by a problem with the revaluation formula/process as such are rarer. That is why it is more precise to speak of a ‘valuation system error’ rather than ‘model error’.